Industrial QUALITY CONTROL

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In This Issue

Application of Consolidated Computer to the Solution of a Metallurgical Regression Problem
—Donald S. Leckie & Richard F. Cain

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—Acheson J. Duncan

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—David J. Blackwell

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—Bernard Giffler

Some Applications of Hotelling's T
—Charles R. Hicks

QUALITY
through
Scientific Methods

Journal of the
AMERICAN SOCIETY FOR QUALITY CONTROL

Industrial Quality Control

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Table of Contents

ARTICLES

Application of Consolidated Computer to the Solution of a Metallurgical Regression Problem
Donald S. Leckie & Richard F. Cain 3

Chi Square Tests of Independence and the Comparison of Percentages
Acheson J. Duncan 9

The Challenge of SQC in Manufacturing Plastics Raw Materials
H. L. Wehrly 14

Acceptance Sampling
David J. Blackwell 17

Simplifying Quality Control Operations with Marginal Punch Cards
Bernard Giffler 20

Some Applications of Hotelling's T
Charles R. Hicks 23

DEPARTMENTS

Problems Department
Paul C. Clifford and Thomas A. Budne, Co-editors 27

Practical Aids, Ellis R. Ott, Editor
Trouble-Shooting
Ellis R. Ott 27

Bibliography
Joseph Movshin, Editor 29

What's New?
Roy Wylie, Editor 30

Local Section News 34

Classified Ads 40

Consulting Services 40

Index to Volume XI 41

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IT PAYS . . .

Good delivery of an idea depends on several things, not the least of which is personal opinion. Some people feel that a thought should be developed, modified, reworked and finally banged home as the climax of the delivery—that is the presentation of the idea should be in much the same sequence as its development—leaving the punch line until last.

Others, especially businessmen, are of the opinion that effective presentation of a thought must be made so that they are hit with the principal idea first. This gives them a chance to look further, if interested; reject it, if not. In addition the main idea coming first establishes a frame of reference so that the further development has meaning.

While this writer is generally opposed to the speeded-up techniques of modern presentation (which frequently serve as an excuse for slipshod methods), let's concede this point to those who are working to a tight time budget and say right off that the subject of this and a few editorials to come is: ADVERTISING.

Now having tipped our hand, those who don't really need to will continue on because of interest in the subject. Those who should continue will probably see little connection between quality control and advertising and be tempted to leave. Stick around awhile; you might be pleasantly surprised.

Advertising as a general subject is, of course, so broad that we must put some limits on the discussion. Actually we propose to discuss advertising from about three different viewpoints: first from the standpoint of each of us as consumers, next in relation to each of us as QC engineers with our respective companies, and finally as related to the fact that we are members of ASQC.

As members of the general consuming public we are all aware that advertising plays an important role in our individual lives. Much advertising is good, does not insult our sensibilities, and hence is something to which we will and do pay attention. A fair share, while not actually bad, is so mediocre that it has no effect on us one way or the other. And a small portion, but unfortunately still too large, is actually so insulting to the intelligence and human dignity as to have an effect exactly opposite to that desired.

What is the purpose of advertising? Two related ideas are fundamental. As an economic function the basic purpose of consumer advertising is to raise our standard of living by creating wants within us which cause us to buy more goods. Consider, for example, the role of advertising over the past 25-30 years in getting one car into every family. Advertising people are currently embarking on a campaign for two cars in every family and they'll probably make it sooner or later. The other purpose is more direct, but equally basic, and that is to sell more products for the particular manufacturer doing the advertising. Frequently individual manufacturers will get into an advertising battle trying to increase their respective proportions of the total market. Occasionally this will degenerate into a dog fight, but much more often it results in a general industry-wide improvement which can only benefit us as consumers.

We all know, for example, that TV as a new medium for advertising is pretty fine. We also know that too many of the TV advertising skits border on the nauseating. This does not mean that *ergo* all TV advertising is bad. In fact those who categorically denounce all advertising because of the few rotten apples are missing out on a very valuable source of information.

The truth is that good advertising is a beneficial social force. Over and beyond its commercial value to the advertiser, it has its place in our magazines, just as does the editorial matter. Take a little closer look at the advertising in your next few issues of Post, Colliers or Readers Digest. Didn't you learn something new and worthwhile that you wouldn't have otherwise learned?

—GRF

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ASQC Publications Office
George R. Foster, *Managing Editor*
Room 6123 Plankinton Bldg.
161 W. Wisconsin Ave.
Milwaukee 3, Wis.

Advertising Representatives

Eastern: Weston and Weston
6 East 39th Street
New York, N.Y.

Midwest: L. B. Hammond
30 North LaSalle Street
Chicago 2, Ill.

★ ★ ★ ★ ★

Society Headquarters Office
Room 563
50 Church Street
New York 7, N. Y.

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C. E. Fisher, *Executive Secretary,*
American Society for Quality Control
Room 563
50 Church Street
New York 7, N. Y.

Make checks payable to:
American Society for Quality Control.

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Application of Consolidated Computer to the Solution of a Metallurgical Regression Problem

DONALD S. LECKIE RICHARD F. CAIN

Republic Steel Corporation

OBJECTIVES OF PAPER

The use of the methods of correlation analysis in the investigation of industrial problems has become of increasing interest in recent years. The methods themselves are not particularly new; many sciences have used them in one way or another for a long time. Industrial "trouble-shooters", however, are now adapting the more advanced analytical techniques to supplement their established methods of data analysis.

One might ask, "What information does the application of multiple correlation (or multiple regression as others call it) to a set of data furnish to the investigator?" Answered briefly, the method develops an equation with which some quantity, or variable, can be estimated, and a measure of the closeness of the estimates to the actual values can be computed. The method also yields a set of factors which indicates the expected change in the dependent variable with a given change in each of the included independent variables. The reliability of conclusions may be estimated to the extent that the data are representative of the usual conditions affecting the process. If the data represent a special set of conditions any conclusions will be valid for that set.

It may properly be suspected that the development of these equations is a long, tedious process if several variables are to be included with enough data to be representative of the many conditions affecting a process. The purpose of this paper is to show how we have been able to reduce many of the tedious mathematical computations to a routine, in order that the time of the investigator may be more profitably spent in planning the investigation, in interpreting the results, and in checking predictions and estimates, rather than chasing computational mistakes and sweating out answers.

Specifically, we shall show how a combination of IBM computing equipment and the Consolidated Computer has been used to transform the handling of data to routine machine operations. To illustrate this, a problem has been chosen which permitted predictions in the interpretation phase, and furnished subsequent checks on the accuracy of the predictions. This problem involves the study of the production rate of a single furnace in one of our open hearth shops. The effects of variations of a number of operating variables upon tons per hour as a measure of open hearth performance were being sought.

This specific study is not considered to be an ideal textbook model of a correlation study. It was thought that it would be more useful to present a study where compromises with the ideally desirable have been made to illustrate the problems which face an investigator when he undertakes such a practical study. An example is the use of tons per hour as a dependent variable. Unquestionably, there are many instances where the use of a rate, tons per hour for example, as a dependent variable will yield helpful information to the investigator, especially when the rate is indicative of the "speed" of a more or less continuous production process.

However, consider the way in which steel usually is produced from an open hearth furnace. Limestone, ore, and scrap are charged into the furnace and melted down. Hot metal from the blast furnace is added, and then the entire batch is refined for several hours before the steel is tapped from the furnace. Both tons produced and heat time (elapsed time from the start of charging through tapping to the start of charging for the succeeding heat) have large, relatively independent variations. Tons per hour is computed by dividing tons produced by heat time, consequently tons per hour is known exactly when the other two factors are known. This ratio in no way indicates that the furnace did produce a certain number of tons of steel within any specific hour.

A more direct approach in the correlation procedure would be to study the effect of operating variables on both tons produced and heat time as dependent variables. This could be done because extensive experience with this type of investigation has seldom indicated the existence of a strong relationship between tons produced and heat time for the usual range of heat sizes made in a given furnace.

It should be noted that since tons per hour is a ratio of two variables in this specific case, we shall be fitting a linear model to a dependent variable which is actually non-linear with respect to some of the included independent variables. Tons per hour, however, is a traditional measure which open-hearth operating people are psychologically prepared to discuss. Also it has the effect of combining into one variable the two quantities, heat size and heat time. The foregoing remarks have been made to forewarn the investigator that when ratios are involved in correlations, caution is required in interpretations.

When we include scrap charging time as an independent variable we cannot help but have a resulting correlation, because heat time is the sum of this specific time period plus others. We desire, however, an estimate of the relationship between the dependent variable and other independent variables with the effect of variation of scrap charging time excluded, and we achieve this effect by including scrap charging time as an independent variable.

In this presentation, therefore, we shall claim neither that this is the ultimate in open hearth studies nor that we have developed a universal method to be applied in all cases. Nor do we present the conclusions of this study as the optimum in open

hearth operating practice. We can, however, show that our methods in this case enabled us to make predictions which were reasonably borne out in subsequent practice. In this case, at least, our compromises seem justified by results.

PLANNING THE STUDY

Several questions arising from discussions of previous studies and shop practices suggested a study, by multiple correlation, of a rather large number of operating variables to determine what variations in furnace practice are most responsible for large variation in tons per hour from heat to heat. The development of effective controls of the more influential variables with consequent improvement in furnace performance was envisioned.

Considerable careful planning with open hearth supervisory and technical personnel preceded the gathering of data. Special reports forms were devised to facilitate the reporting of data not included in routine heat records. Operating personnel, such as melters, furnace helpers, stockers, and recorders were briefed on the aims of the study and the importance of accurate data. It was decided to study the performance of one 190-ton gas and oil-fired open hearth furnace for approximately the extent of a furnace campaign. This furnace normally charged scrap and about 40% hot metal and produced various grades of carbon and alloy steels.

GATHERING THE DATA

A list was compiled of operating variables which included those considered to be important and others thought to have possibilities of importance. The data applying to such variables were reported and tabulated by open hearth personnel under the supervision of the Quality Control Department. In the interest of some economy of effort, the 23 variables in Table I were selected for correlation. These were discussed with the operating people who agreed that the most important and most promising measurable variables were included. Under other conditions and with other aims different variables might be used. Exceptional heats (roof patches, forced tap, etc.) were omitted leaving 125 heats produced over a period of about three months. Since no separation was made for grade, the data represent the average range of grades generally made in the furnace over a period of time.

TABLE I—Operating Variables Selected for Investigation

Dependent Variables

1. Tons per Hour
2. Time from Finish Charge Hot Metal to Block

Independent Variables—Group I

1. Heat Size
2. Time from Start Charge to Finish Charge Scrap
3. Length of Lime Boil Period
4. Time from Tap to Start Charge of Following Heat
5. Per Cent of Scrap in Initial Charge
6. Average Weight of Scrap per Buggy
7. Weight of Limestone in Initial Charge
8. Boxes of Burned Lime Added During Heat
9. Per Cent Hot Metal Charged
10. Per Cent Silicon in Hot Metal
11. Carbon Drop from End Lime Boil to Block

Independent Variables—Group II

1. Fuel Rate from Start Charge to Charge of First Hot Metal
2. Fuel Rate from Charge First Hot Metal to Start Lime Boil
3. Fuel Rate During Lime Boil Period
4. Fuel Rate from End of Lime Boil to Block
5. Steam Consumption per Gallon of Fuel Oil
6. Atmospheric Humidity
7. Carbon Oxidation Rate from First Test to End of Lime Boil
8. Lime/Silica Ratio of Slag at Block
9. Per Cent FeO in Slag at Block
10. Bath Temperature at Block

PREPARATION OF DATA FOR COMPUTER

After a set of data has been accumulated, an investigation enters the cumbersome computational stage. To keep such an investigation from dying of its own inertia, these calculations must be done quickly and accurately for top efficiency. Accuracy must be stressed especially. The most disheartening task for the investigator is to seek mistakes after inconsistencies have become apparent in the computations. Also, unless there is absolute assurance of accuracy, the nagging suspicion will always be present that a mistake may have been made which did not result in any inconsistency observed during the process of calculation.

Since it is beyond the scope of this paper to present the details of a multiple correlation problem, from this point on reasonable familiarity with such procedure must be assumed.

The first step is to code the data to a suitable number of digits so it can be handled easily in the calculating machines. Our practice is to code to a maximum of three digits, although often two digits will prove to be satisfactory. Coding may be done by adding or substracting a constant, or by multiplying or dividing by a constant, or both. The smaller the coded numbers are, the smaller the resulting squares and cross-products will be, thus providing more convenient numbers for future computations. Care must be taken, however, not to code data so coarsely that significant variation is lost. For instance, tons per hour could be rounded to the nearest unit. This would, however, eliminate the tenths which are certainly significant, since tons and heat time are routinely reported to three significant figures. Consequently, tons per hour were coded (Table V) by multiplying the column of data by ten to get whole numbers, then subtracting 70, to reduce the size of the coded numbers. All coding must be accurately done or the investigator will be analyzing a set of data which is different from the one which was so carefully collected.

The coded data for this particular study were tabulated in 23 columns on ruled sheets of paper, and then these sheets were given to the Tabulating Department where the operators key-punched the data into cards on an IBM No. 024 Numerical Key Punch. The cards were set up to contain 23 three-digit fields, or 69 digits. This master deck of cards was verified for correct punching and

Table II—Summary of Sums of Columns, Sums of Squares, and Sums of Cross Products—Ten Important Variables

	X_0	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
Sum of Cols.	6144	4140	5421	6209	4202	1520	523	3803	3712	3213	5404
Sum of Squares	334,582	150,062	334,507	333,301	155,968	34,024	3,609	135,471	113,316	101,797	241,782
Cross Products											
X_0		208,091	259,850	294,718	204,825	60,459	23,445	182,882	182,619	160,581	269,838
X_1			181,342	203,882	139,180	50,373	18,019	129,649	122,770	105,052	180,130
X_2				266,307	186,586	65,360	27,501	150,565	162,781	141,286	237,831
X_3					202,219	80,449	26,019	187,098	186,456	161,505	262,920
X_4						49,230	18,322	131,296	123,805	105,943	184,563
X_5							6,767	45,259	45,807	38,375	63,847
X_6								16,206	15,848	13,546	22,612
X_7									111,741	96,018	165,121
X_8										96,037	160,632
X_9											138,664

Table III—Example of Tabular Form for Computing the Set of Simple Correlation Coefficients (r's)

Col. 1 Variables X_a—X_b	Col. 2 N	Col. 3 $\Sigma x_a x_b$	Col. 4 Σx_a	Col. 5 Σx_b	Col. 6 $N\Sigma x_a x_b - \Sigma x_a \Sigma x_b$	Col. 7 L_a	Col. 8 L_b	Col. 9 $L_a L_b$	Col. 10 r_{ab}
X_0—X_1	125	208,091	6144	4140	+ 575,215	2018.42	1272.08	2,567,592	+ .2240
X_0—X_2	125	259,850	6144	5421	— 825,374	2018.42	3525.07	7,115,072	— .1160
X_0—X_3	125	294,718	6144	6209	— 1,308,346	2018.42	1763.78	3,560,049	— .3675
X_0—X_4	125	204,825	6144	4202	— 213,963	2018.42	1356.16	2,737,300	— .0782
X_0—X_5	125	60,459	6144	1520	— 1,781,505	2018.42	1393.77	2,813,213	— .6333
X_0—X_6	125	23,445	6144	523	— 282,687	2018.42	421.42	850,603	— .3323
X_0—X_7	125	182,882	6144	3803	— 505,382	2018.42	1571.96	3,172,876	— .1593
X_0—X_8	125	182,619	6144	3712	+ 20,847	2018.42	620.93	1,253,298	+ .0166
X_0—X_9	125	160,581	6144	3213	+ 331,953	2018.42	1549.60	3,127,744	+ .1061
X_0—X_{10}	125	269,838	6144	5404	+ 527,574	2018.42	1009.76	2,038,120	+ .2588
Sum & Check	125	1,847,308	6144	38,147	— 3,461,668	2018.42	14,484.53	29,235,867	
X_1—X_2	125	181,342	4140	5421	+ 224,810	1272.08	3525.07	4,484,171	+ .0501
X_1—X_3	125	203,882	4140	6209	— 220,010	1272.08	1763.78	2,243,669	— .0981
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.

The simple correlation r_{ab} between any two variables, X_a and X_b, is computed from the formula:

$$r_{ab} = \frac{N \Sigma x_a x_b - \Sigma x_a \Sigma x_b}{\sqrt{N \Sigma x_a^2 - (\Sigma x_a)^2}\ \sqrt{N \Sigma x_b^2 - (\Sigma x_b)^2}}$$

The Quality Control Department computes L_a and L_b on a desk calculator:

$$L_a = \sqrt{N \Sigma x_a^2 - (\Sigma x_a)^2}$$

$$L_b = \sqrt{N \Sigma x_b^2 - (\Sigma x_b)^2}$$

then run through the IBM No. 402 Tabulator to sum the columns and record the sums. These sums were checked against a set of column totals obtained on an adding machine in the Quality Control Department as a final check on the key-punching.

The Tabulating Department has a method of computing the sums of squares and cross-products of columns by duplicating the master deck on an IBM No. 514 Reproducing Punch Machine and then running the decks through either a No. 602 or a No. 604 Multiplier. The multiplier accumulates the sums of the squares and cross products as a deck runs through the machine, and the accumulated answer is punched in a summary card following the deck. The multiplication is done in such a way that each sum of squares and cross-products is calculated in two independent ways for checking. The summary cards are then tabulated on an IBM No. 402 Tabulator for a listing of the sums of squares and cross-products as well as the set of checks. A summary of the sums, squares, and cross-products of 11 important variables is included in Table II.

At this point some people immediately set up and proceed to solve the normal equations. Others, however, compute simple correlation coefficients (r's) from the preceding data and use these r's in the normal equations. There are certain mechanical objections to the direct use of squares and cross-products. An example is the number of significant figures that must be carried to preserve the required accuracy in the solution. Also, the computation of auxiliary information such as R^2 (the multiple coefficient of determination) is simpler with the data transformed, so we have always considered the extra operation worth while.

TABLE IV—Simple Correlation Coefficients (r's) for the Ten Independent Variables Selected as Most Important

	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_0
X_1	+ .0501	— .0981	+ .0007	+ .0022	+ .1626	+ .2309	— .0271	— .0864	+ .1119	+ .2240
X_2		— .0596	+ .1138	— .0142	+ .4055	— .3240	+ .1027	+ .0445	+ .1219	— .1160
X_3			— .3398	+ .2516	+ .0068	— .0814	+ .2367	+ .0873	— .3865	— .3675
X_4				— .1234	+ .1620	+ .2025	— .1451	— .1228	+ .2649	— .0782
X_5					+ .0867	— .0562	+ .0966	— .0402	— .1657	— .6333
X_6						+ .0555	+ .1514	+ .0196	+ .0004	— .3323
X_7							— .1528	— .0890	+ .0559	— .1593
X_8								+ .0810	+ .0309	+ .0166
X_9									— .0192	+ .1061
X_{10}										+ .2588

For a large set of data such as this the calculation of the r's is a routine operation adaptable to IBM equipment. Table III displays the formulas and the tabular form used. The Quality Control Department fills in Columns 1, 2, 3, 4, 5, 7, and 8 on the form. Cards are punched from these data, and the IBM Department computes the numerator and the denominator product of the formula in Columns 6 and 9. The division of Column 6 by Column 9 may be performed by IBM equipment or on a desk calculator.

The calculation of r's is done by groups, with the first group containing the correlation of a selected variable with all the others, the second containing the correlations of a second variable with the other variables less the first selected variable, etc. The computation may be checked by summing the columns and performing the calculations as indicated in Table III. The result of applying these calculations to our data is shown in Table IV which lists the r's to be used for the solution of the normal equations.

In some textbooks the emphasis at this point is on the direct solution for the coefficients by some method such as Doolittle's or Dwyer's. For explaining the operation of the computer, however, the simultaneous solution of equations will have to be stressed. These equations are of the form:

$$\begin{aligned}
&\beta_1 + r_{12}\beta_2 + r_{13}\beta_3 + \ldots + r_{1N}\beta_N = r_{01} \\
&r_{12}\beta_1 + \beta_2 + r_{23}\beta_3 + \ldots + r_{2N}\beta_N = r_{02} \\
&\quad\vdots \\
&r_{1N}\beta_1 + r_{2N}\beta_2 + r_{3N}\beta_3 + \ldots + \beta_N = r_{0N}
\end{aligned}$$

The r's are known and the Betas are the unknowns whose solutions are required. For n independent variables there are n unknown Betas and n equations. Any one who has tried to solve four or more simultaneous equations algebraically has some idea of the extent of the tedious calculations. The Consolidated Computer solves such equations by the entirely different method of successive approximations. This machine is the Model 30-103 Computer manufactured by the Consolidated Engineering Corporation, Pasadena, California.

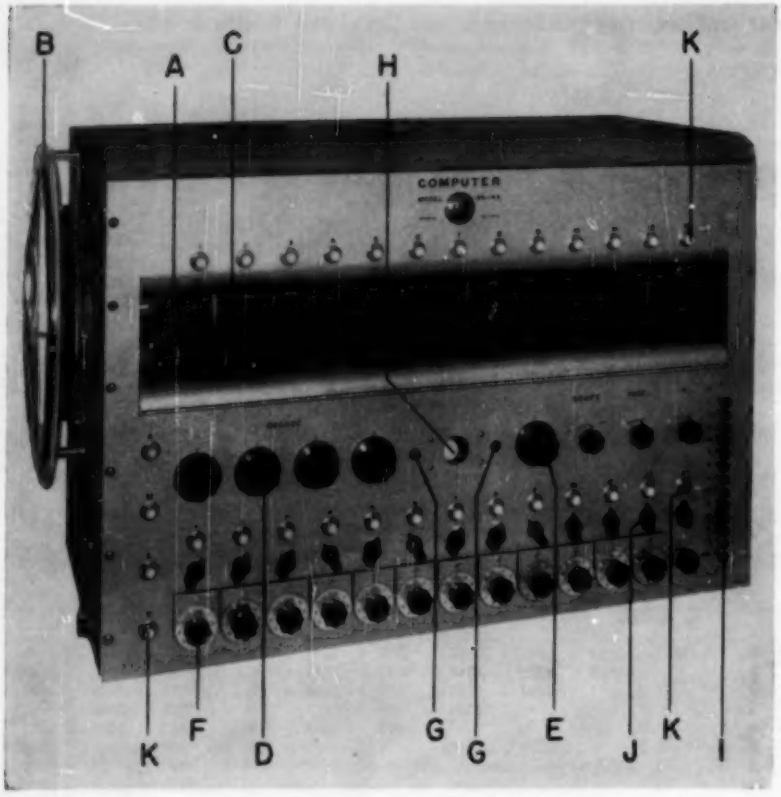

FIGURE 1

A — Turret
B — Hand Wheel
C — r Coefficient Helipots
D — Decade Switch — 4 digits
E — Column Selector Switch
F — Beta (Unknown) Helipots
G — Neon Glow Lamps
H — Cathode Ray Tube
I — Polarity Push Buttons
J — Polarity Switches
K — Pilot Lights

DESCRIPTION OF COMPUTER

A diagrammatic picture of the front panel is shown in Fig. 1. This computer is an analog, rather than a digital, type. The computer houses a large turret (A) which contains twelve rows of 13 helipots (C) (potentiometers) each, and which is revolved to the proper row at the desired time by the hand wheel (B). The coefficients of each equation (r value) are set on the helipots in an equivalent row on the turret. Thus, the machine has a capacity of 12 linear equations with 12 unknowns. A system of pilot lights (K) indicates which circuit components are connected for various settings of the operating switches.

The r coefficients are set on the computer by means of the decade switch (D) which has four dials, each numbered from zero to nine. Any four digit number may be set on the dials and there is a special provision for setting the number 1.0000. Since we have always followed the practice of computing the correlation to four decimal places the machine has the proper capacity for our use. We have found in the past that r's computed to more than four decimal places do not seem to increase desirably the accuracy of the solution in view of the possible inaccuracies in the basic data. If computed to less than four places the resultant accuracy may sometimes be less than desired.

Any selected r is entered in the machine by revolving the turret to the proper row and by setting the column selector switch (E) to the proper column. The number is set on the decade switch and then the proper helipot, indicated by pilot lights, is turned in the direction indicated by the neon glow-lamps (G) until a circular pattern on the cathode ray tube (H) closes to a horizontal line. The r's for the dependent variable are always set in the last column of helipots on the right-hand side of the turret.

After the entire set of equation coefficients has been set up on the turret it is possible to check them easily. The computer is now ready to solve for the Betas. These Beta values are to be found on the bottom row of helipots (F).

The solution is achieved by a method of successive approximations, the mathematical explanation for which is considered to be inappropriate here. The computer is constructed so that the products of the r's on the turret and their respective Betas are summed and compared with the dependent r's which are on the right of the equal sign in the equations. The Betas are adjusted to balance the equation as indicated by closure of the circle on the cathode ray tube. Repeated approximations are made until a set of Betas is obtained which balances all equations. This set is the required solution of the equations. The solution is obtained in orderly fashion and successive approximations result in succeedingly smaller corrections to the Betas.

The computer is equipped with a set of polarity push buttons (I) and a set of polarity switches (J). These are both necessary to set the correct signs of the r coefficient for each equation as it enters the computer circuit during the solution cycle. The values of the Betas are found by balancing the decade switch against the Beta helipots, which may be selected in turn by the column selector switch. The Betas can be given an auxiliary check by substitution in any or all of the original equations on a desk calculating machine.

The literature [1] with reference to the computer gives instructions for arrangement of the data for optimum speed of convergence to the solution. Under ordinary operating conditions, the coefficients for a twelve independent variable problem can be set up and checked in approximately 45 minutes and the solution may take from 10 to 45 minutes, depending on speed of convergence. This is quite a saving in time over the usual computational methods. An occasional problem will require an hour or more for solution, but such problems generally contain some statistically or mathematically undesirable condition, such as high intercorrelation among the independent variables.

One of the most valuable features of the computer is that once the r's are set up on the computer, different combinations of the independent variables may be studied by omitting the proper columns and equations in the solution. Also, a different dependent variable may be conveniently substituted on the turret to

TABLE V—Summary of Important Statistics Derived from Correlation Study of the Ten Most Important Independent Variables

Variable	Units	Code	$\bar{X}$	σ	r	Beta from Computer	Beta from Doolittle Sol.	t-Test	B
X_0 = Tons per Hour	T/Hr.	x=10X—70	11.92	1.615					
X_1 = Heat Size	Tons	x=X—150	183	10.18	+ .2240	+ .3141	+ .3139	5.98	+ .0499
X_2 = Carbon Drop from End Lime Boil to Block	%	x=100X	0.434	0.282	— .1160	— .1563	— .1560	2.61	— .8951
X_3 = Time from Start Charge to Finish Charge Scrap	Hrs.	x=10X	4.97	1.411	— .3675	— .2676	— .2677	4.58	— .3063
X_4 = % Scrap in Initial Charge	%	x=X	33.62	10.85	— .0782	— .1179	— .1179	2.10	— .0175
X_5 = Time Between Heats + Delays	Hrs.	x=6X	2.03	1.858	— .6333	— .5704	— .5703	11.03	— .4958
X_6 = Burned Lime Added	Boxes	x=X	4.18	3.371	— .3323	— .2557	— .2558	4.45	— .1225
X_7 = Time of Lime Boil	Hrs.	x=12X	2.51	1.048	— .1593	— .2749	— .2747	4.80	— .4236
X_8 = % Hot Metal	%	x=X—10	39.70	4.967	+ .0166	+ .1285	+ .1286	2.43	+ .0418
X_9 = Carbon Oxidation Rate from 1st Test to End Lime Boil	%	x=100X	.257	0.124	+ .1061	+ .0981	+ .0980	1.95	+1.2777
X_{10} = Oil Rate from Charge 1st Hot Metal to Start Lime Boil	Gal./Hr.	$x=\frac{1}{5X-40}$	416	40.39	+ .2588	+ .0894	+ .0893	1.61	+.00357

Estimating Equation:

$$X_0' = + .0499\,X_1 - .8951\,X_2 - .3063\,X_3 - .0175\,X_4 - .4958\,X_5 - .1225\,X_6 - .4236\,X_7 + .0418\,X_8 + 1.2777\,X_9 + .00357\,X_{10} + 4.41$$

Standard Error of Estimate of X_0:

$S_{X_0} = 0.852$

$R^2 = 0.7217$

$R = 0.8495$

Significance Levels of t

$t_{.01} = 2.62$

$t_{.05} = 1.98$

$t_{.10} = 1.66$

$$B_a = \frac{\sigma X_0}{\sigma X_a}\,\beta_a$$

solve any combination of the independent variables with a new dependent variable. Regarding the set of correlations of the independent variables as a square matrix, the inverse of this matrix can be computed and the errors of the Betas computed from the diagonal terms. These are a few representative examples of the versatility and the value of the machine.

CALCULATION WITH THE COMPUTER

We have said that the machine has a capacity of 12 independent variables, and we have indicated that we had 23 variables in our set of data. This certainly exceeded the capacity of the computer. There are methods for extending the capacity of the computer to accommodate more than 12 variables. In this case, however, we did not try to solve with all 21 independent variables simultaneously, but used the following method:

The 21 independent variables were divided into two groups, one of 11 commonly studied variables whose effects had been studied before, and the other ten variables which had not been studied before because of a lack of adequate data, but in whose effects we were especially interested.

The first group was correlated with tons per hour as the dependent variable and the resultant coefficient of multiple determination, R^2 was 0.7087. Using a test developed by R. A. Fisher[2] the three weakest variables with respect to contribution to R^2 were eliminated. The new solution with only eight independent variables yielded an $R^2 = 0.7058$, only slightly lower than before. We then concluded that in this group the eight selected were the most important, and reserved them for further study.

When the second group was tested in a similar manner, all ten independent variables yielded a low $R^2 = 0.1624$, and removing five of the ten variables resulted in an $R^2 = 0.1508$. The remaining five, together with the eight selected from Group I, gave a total of 13 variables and these were studied by using the eight from Group I and varying combinations of four of the five from Group II. It was finally determined that only two of the five from Group II were significant and the resulting R^2 with ten independent variables was 0.7217.

Thus, ten of the 21 independent variables indicated that they were important enough in their effect upon tons per hour to merit further investigation and attempts toward increased control. It should be stressed that no conclusion can be made that the discarded 13 are totally unimportant in their effects upon open hearth production. The seeming lack of relationship may be due to limited ranges of variation in the practices during the data period, errors arising from sampling of tests, inadequate measurements, etc. Some of these relationships could be found, perhaps, with an experiment planned specifically to measure a particular variable effect.

Table V summarizes the pertinent results of this study and lists the simple correlations of the several independent variables with the dependent variable (r's), the Beta values, the t-Tests of Significance of the Betas, the R^2, and the Standard Error of Estimate (Sx_o) of the dependent variable. For comparison, the Betas calculated by the Computer and by the abbreviated Doolittle method are shown. Note that the agreement is close between pairs of Betas computed by the two methods. The Betas from the computer solution are of sufficient precision to be used for estimating purposes, with results well within the possible errors in the original data. The t-Test given is the ordinary statistical test of significance and is used to determine if the coefficient is so different from zero that it is unlikely to be that large due to random causes alone. The one percent and five percent significance levels are given and may be compared in the usual way. A majority of the variables will be seen to be highly significant. The estimating equation is also shown. This completes the laborious computation section of a study and, if the calculations are free of mistakes, the equation yields the least-squares estimates of the dependent variable with a predicted error or variance.

INTERPRETATION OF RESULTS

The results were now ready for the very critical phase of interpretaton and presentation to operating personnel. In many industrial applications, doubts of the multiple correlation technique have arisen due to insufficient care in interpretation. In our organization, before interpretations are released to operating supervision, a review panel carefully evaluates the study both statistically and also from the standpoint of practical experience and prior knowledge. Conclusions which

may not make sense at first sight are studied to determine if they might be possible under certain conditions, if they might have appeared due to an omitted factor, or if they might be due to incorrect data, etc. It should be stressed that we never immediately discard this type of statistical result merely because it fails to make sense or conform to previous opinion. It will always be possible to find new relationships in most processes, and many times it is desirable to collect more data and experiment further to check such an "absurd" conclusion for repetition.

It should always be recognized that the relationships fitted to the data by this technique have been linear, although in many cases the relationships are known to be curvilinear. The linear estimates, however, are a first approximation, and frequently the curvature is so slight in the range of the variable studied that the linear approximation is a very good one. Methods are available in the literature for investigation of curvilinearity after the problem has reached this stage. Also, it is important to remember that simple correlations and even multiple correlation results do not necessarily yield cause and effect relationships. Sometimes these relationships may be symptomatic as, for example, the significant relationship of the variable burned lime added with tons per hour, which reflects a condition of the slag-bath chemistry. Any "treatment" must apply to the condition of the slag-bath rather than to the symptom (burned lime added.) In any event, cause and effect is demonstrated only when the results of actual process changes lie within the region of predicted results.

After the results of the correlation were digested by the reviewing panel, a meeting was held with open hearth operating supervision which included all foremen. Instead of presenting the mathematics and formulas, the results were presented by listing the probable effects on tons per hour for an attainable change in the average of important variables. In total, the changes recommended for the variables indicated a probable increase of 0.99 tons per hour from an average of 11.92 to 12.91. Some typical conclusions were:

(a) Increasing average heat size five tons per heat would mean a probable gain of 0.250 tons per hour.

(b) A reduction of one-half hour in scrap charging time would mean a probable increase of 0.153 tons per hour.

(c) A decrease of 20 minutes of non-productive time (bank and bottom-making delays) would mean a probable increase of 0.166 tons per hour.

(d) The study had shown that an average of 4.2 boxes of burned lime was added during the heat and if this amount were reduced by two boxes, on the average, the probable gain would be 0.241 tons per hour. To be able to do this, the amount of limestone charged at the start of the heat would possibly have to be increased, but the study indicated that this probably would have a negligible effect upon tons per hour.

The indicated potential increase in production rate was of sufficient size to be highly attractive to supervision who were readily induced to attempt the recommended changes in practice which might accomplish this improvement.

It must be noted that much of what we learned from the correlation was not news to anyone familiar with the process. The value of the study lay in the numerical approximations and estimates of the relationships, by which the effect of changes could be predicted quantitatively. These numerical relationships from the multiple correlation were free from the effect of variations of other included variables, to say nothing of personal opinions. In this manner, any increased cost affecting the proposed alterations in the independent variables could be evaluated against the advantages of the increased tons per hour which might be expected to result.

FOLLOW-UP STUDY OF SUGGESTIONS

Since a correlation measures associations without regard to their cause-effect quality, it should be checked in practice. It has little value if its indications cannot be so checked to prove the validity of the cause-effect assumptions. The justification of the correlation equation is its successful prediction of the effects of changes in practice.

Such a check was made by comparing the actual change of average tons per hour, over a four-month period following the meetings with open hearth personnel, with the predicted change. The predicted change was calculated by substituting in the estimating equation the actual values obtained during this subsequent period for those independent variables on which data were routinely obtained. Data for 177 heats were collected from routine heat records. Unfortunately, data for some variables used in the equation are not routinely reported, so a complete check of the equation is not available. Grade distribution, average tons tapped, and average non-productive time (delays and bottom time) remained approximately the same during both periods. Other unmeasured variables have to be assumed not to have changed so as to appreciably affect tons per hour.

Certain of the suggestions resulting from the interpretations of the correlation were followed in part, while others were modified due to existing shop conditions. Improvement to the extent of 0.70 tons per hour resulted. While this falls somewhat short of the predicted increase of 0.99 tons per hour, the difference has been critically examined and is felt to be reasonably explainable. For example, there is the known nonlinear nature of the scrap charging time regression line. Moreover, since data for five of the ten independent variables in the equation are not routinely reported, the estimate may be considered to be reasonably close, and the assumptions of cause-effect relationship appear to have been sufficiently accurate for practical use.

Credit for improvement is not claimed by the Quality Control Department. Its function is to interpret the data to the open hearth people in such a way that they may take whatever action they can to benefit themselves. It also had the means of measuring the improvement against a standard—the estimating equation.

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Chi Square Tests of Independence and the Comparison of Percentages

ACHESON J. DUNCAN

The Johns Hopkins University

In industrial research and experimentation it is often important to determine whether certain factors or certain conditions "interact" to affect the results or whether they act independently of each other. When the data in hand are frequencies of occurrence; i. e., are attributes or enumeration data in contrast to variables or measurement data, the statistical technique commonly employed to determine the existence of interaction is the χ^2 test of independence. It is the purpose of this paper to explain briefly the how and wherefore of this technique and to illustrate some of its uses. (2)

EXAMPLE OF INDEPENDENCE TEST

Let us begin by considering the data of Table I. These pertain to failures of vacuum tubes used during World War II in VT fuses. The data were derived in the course of attempts to standardize test conditions for the fuses and provided an answer to the question: Is the distribution of tubes by type of failure the same when the tube is located in the top block of the shell as when it is placed in the bottom block? In other words, is type of failure independent of position in the shell or is there an interaction between them?

In general a test of independence of two such factors consists of two steps:

(1) determining how the data would be expected to be distributed if the two factors were independent, and

(2) comparing the actual distribution with the expected.

Let us illustrate the first step with respect to the data of Table I. If the distribution of type of failure is independent of the position of the tube in the shell, we would expect the *proportions* of the tubes falling in categories A, B, and C to be the same for the top block as for the bottom block of the shell. In other words, we would expect the frequencies in the row marked "Top Block" to be *proportional* to the frequencies in the row marked "Bottom Block," the factor of proportionality being the ratio of total failures in the bottom block to total failures in the top block. Another way of saying the same thing is that we would expect the 115 type A failures to be distributed in the same proportion as the total failures, viz., in the ratio of eight tubes in the bottom block for each ten tubes in the top block. Likewise, we would expect the 40 type B failures to be distributed between bottom and top blocks in the ratio of eight to ten, and also the 25 type C failures. The distributions expected on the hypothesis of independence would, therefore, be those given in Table II.

The data of Table I do not conform exactly to those of Table II. The data of Table I are sample data, however, and allowance must be made for sampling fluctuations. What we would like to know is whether the results of Table I differ sufficiently from those of Table II to make the hypothesis of independence untenable. To put it another way, is such a sample result very unlikely if we assume the existence of independence and allow for sampling fluctuations? It is to answer such questions that the χ^2 test is employed.

A χ^2 test consists of comparing actual and expected frequencies. The formula used is

$$\chi^2 = \Sigma\left[\frac{(F-f)^2}{f}\right] \qquad (1)$$

where F is the actual frequency of any cell, f is the frequency expected on the hypothesis of independence, and the sum is taken over all cells. For Tables I and II, this formula yields the following:

$$\chi^2 = \Sigma\left[\frac{(F-f)^2}{f}\right]$$

$$= \frac{(75-63.9)^2}{63.9} + \frac{(10-22.2)^2}{22.2} + \frac{(15-13.9)^2}{13.9}$$

$$+ \frac{(40-51.1)^2}{51.1} + \frac{(30-17.8)^2}{17.8} + \frac{(10-11.1)^2}{11.1}$$

$$= 1.93 + 6.70 + 0.09 + 2.41 + 8.36 + 0.11$$

$$= 19.60$$

The problem is how to evaluate this result.

According to statistical theory, if we should run many tests of the kind exemplified by Table I (not necessarily keeping the total tubes tested equal to 180, but always

TABLE I—Failures of VT Vacuum Tubes*

Position in Shell	Type of Failure			Total Failures
	A	B	C	
Top Block	75	10	15	100
Bottom Block	40	30	10	80
Total	115	40	25	180

*Reference (3), p. 18.

TABLE II—Frequencies of Failures for the Data of Table I That Would Have Been Expected* on the Hypothesis of Independence, Given the Marginal Totals

Position in Shell	Type of Failure			Total Failures
	A	B	C	
Top Block	63.9	22.2	13.9	100.0
Bottom Block	51.1	17.8	11.1	80.0
Total	115.0	40.0	25.0	180.0

*Expected in the sense of being an average over many samples. This is why the frequencies are not rounded off to whole numbers.

keeping the same classifications and, of course, making a random selection of tubes) and if the hypothesis of independence is actually true, then the sample values obtained by applying formula (1) to each set of sample data would tend to form a special frequency distribution called the χ^2 distribution. (Hence the name Chi-Square Test.) There are various χ^2 distributions, distinction among them being made on the basis of the so-called "degrees of freedom." This term will be explained later. For the moment let it be simply stated that for the problem in hand the degrees of freedom are two. With this in mind, note from Table III that, for a χ^2 distribution with two degrees of freedom, not more than five percent of the sample values of $\chi^2 = \Sigma \left[\frac{(F-f)^2}{f}\right]$ would, if the hypothesis of independence were true, exceed the value 5.99 nor more than one percent exceed the value 9.21.

Suppose now that in making a test of independence we are willing to run a risk of 0.05 of rejecting the hypothesis when it is true.* In such a case, we will take the 0.05 point of the pertinent χ^2 distribution as a critical value for comparison with our sample result. If the sample value of χ^2 exceeds this critical value, we will reject the hypothesis; if not, we will accept it. For the problem at hand, the 0.05 point of the χ^2 distribution with two degrees of freedom is 5.99. The sample χ^2 is 19.60. Hence, we will reject the hypothesis of independence and conclude that type of failure is related to the position of the tube in the shell.

The above explains the technique of carrying out a χ^2 test of independence. Consider now the meaning and determination of the quantity called "degrees of freedom." For the data of Table I, it was said that there were two degrees of freedom. By this it was meant that we had only two independent points of comparison between actual and expected frequencies. Why only two, when formula (1) calls for six comparisons? The answer is that in computing the expected frequencies we made use of the marginal totals, 100, 80, 115, 40, and 25, (as indicated above) in such a way that the actual and expected frequencies in every row and every column added up to the same marginal totals. Consequently we could only determine expected frequencies independently for *two* cells. All the other expected frequencies had to be such that the marginal totals were the same as for the actual frequencies. Hence we had only two "degrees of freedom." In general, if classification I has c categories and classification II has r categories, the degrees of freedom for the χ^2 test will be $(c-1)(r-1)$.

INDEPENDENCE TEST AS TEST OF DIFFERENCE BETWEEN TWO PERCENTAGES

Let us consider now a second example that will illustrate a special application of χ^2 tests of independence and lead to consideration of some of the finer points of the technique. The data that will be studied are those of Table IV. This presents an analysis of automobile tires scrapped as a result of a mileage test. A question of interest is whether the percentage of scrapped tires that came from the right rear side of the car differs significantly from the percentage of scrapped tires that came from the left rear side of the car. This problem illustrates how the χ^2 technique can be applied to testing the difference between two percentages, one of the most important of its applications.

The general technique is the same as before. We first compute the frequencies that would be expected on the hypothesis that the two percentages would, in an infinite universe, be the same, the common percentage being estimated from the totals of rear and all tires. Thus, the proportion of all scrapped tires that were on the rear is $160/400 = 0.4$. If the left and right positions do not "interact" with front and rear positions, we would expect the scrapped rear tires on the right side to be 0.4 of total scrapped right-side tires and the scrapped rear tires on the left side to be 0.4 of the total scrapped left-side tires. In other words, the frequencies that would be expected on the basis of independence would be those of Table V.

When we come to calculate χ^2, however, we find that in this 2 x 2 case it is desirable to make some adjustments in the cell differences before squaring. The improved formula for the 2 x 2 table is

$$(2) \qquad \chi^2 = \Sigma \left[\frac{(|F-f| - .5)^2}{f}\right]$$

TABLE III—The 0.05, 0.01, and 0.001 Points of the χ^2 Distribution*

Degrees of Freedom	Probability of a χ^2 Equal to or Greater Than Listed Value is:		
	0.05	0.01	0.001
1	3.84	6.63	10.8
2	5.99	9.21	13.8
3	7.82	11.3	16.3
4	9.49	13.3	18.5
5	11.1	15.1	20.5
6	12.6	16.8	22.5
7	14.1	18.5	24.3
8	15.5	20.1	26.1
9	16.9	21.7	27.9
10	18.3	23.2	29.6
11	19.7	24.7	31.3
12	21.0	26.2	32.9
13	22.4	27.7	34.5
14	23.7	29.1	36.1
15	25.0	30.6	37.7

*The writer is indebted to R. A. Fisher and Frank Yates, and to Oliver and Boyd, Ltd., Edinburgh, for permission to reproduce the above from Table III of their book, *Statistical Tables for Use in Biological, Agricultural and Medical Research.*

TABLE IV—Distribution of 400 Tires Scrapped After a Mileage Test*

	Front	Rear	Total
Left	115	65	180
Right	125	95	220
Total	240	160	400

*Taken from A. W. Swan, "The χ^2 Significance Test-Expected vs. Observed Results," *The Engineer*, Dec. 31, 1948, p. 679. Since this is an English publication, Table IV reverses the left and right sides so as to bring the results in line with American automobile driving.

TABLE V—Distribution of 400 Scrapped Tires That Would Have Been Expected If the Percentage of Scrapped Rear Tires is Independent of Side Position

	Front	Rear	Total
Left	108	72	180
Right	132	88	220
Total	240	160	400

*In other words the test is being run "at the 0.05 level." It could just as properly be run at a 0.10, or 0.01 or even an 0.001 level. The choice depends on desired risks and costs.

The need for this adjustment arises from the approximate character of the tabular χ^2 probabilities when applied to tests of independence. The exact distributions that apply to these tests are discrete. The tabular χ^2 probabilities are derived from a continuous function that is used to approximate the discrete distributions. In a 2 x 2 table this approximation can be improved by subtracting 0.5 from the absolute differences as indicated above. This adjustment is called "the χ^2 correction for continuity." It should be applied only to 2 x 2 tables and may be omitted if all the cell frequencies exceed 500. Continuity corrections could also be made for r x c tables, but they are somewhat more difficult to apply and are usually deemed unnecessary.

The value of the corrected χ^2 for the given data is

$$\chi^2 = \frac{(|115-108|-.5)^2}{108} + \frac{(|125-132|-.5)^2}{132} + \frac{(|65-72|-.5)^2}{72} + \frac{(|95-88|-.5)^2}{88}$$

$$= (6.5)^2\left[\frac{1}{108}+\frac{1}{132}+\frac{1}{72}+\frac{1}{88}\right]$$

$$= (42.24)\ (.042088) = 1.78$$

Since this is less than the 0.05 point of the χ^2 distribution with one degree of freedom (3.84), we accept the hypothesis that the percentage of rear tires scrapped is independent of their position on the left or right of the car; i.e., the two percentages (% right rear and % left rear) do not differ significantly.

COMPARISON OF PERCENTAGES WHEN SAMPLES ARE VERY SMALL

In the foregoing example, the frequencies ran reasonably large so that the correction for continuity was sufficient adjustment to allow the use of tabular χ^2 probabilities. When the frequencies of a 2 x 2 table run very small, however, say total n is 40 or less, the tabular χ^2 probabilities become rather inaccurate and a direct method of calculation or special tables should be employed that do not use the χ^2 formula.

The exact method may be illustrated by another example. Suppose we are studying the effect of heat treatment on the brittleness of nylon bars and get the results shown in Table VI. The ordinary χ^2 corrected for continuity has the value 5.47 which has associated with it a tabular χ^2 of about 0.021. An exact method based on conditional probabilities is given by R. A. Fisher.* To apply this to the above data, we perform the following computations:

(1) We compute the probability of getting zero out of 11 brittle bars on the assumption that the probability of a brittle bar is p'. $\left(= (1-p')^{11}\right)$

(2) We compute the probability of getting five out of nine brittle bars on the assumption that the probability of a brittle bar is p'. $\left(= \frac{9!}{5!4!}\, p'^5(1-p')^4\right)$

(3) We compute the joint probability of zero out of 11 and five out of nine assuming the two samples to be independent. $\left(= \frac{9!}{5!4!}\, p'^5(1-p')^{15}\right)$

*See Reference 6, 5th and subsequent editions, Section 21.02. Also see Reference 10, Chapter 7, and Reference 4, Appendix 31.

TABLE VI—Effect of Heat Treatment on Nylon Bars

Length of Heating Cycle	Number of Nylon Bars Classed as:		
	Brittle	Tough	Total
30 sec.	0	11	11
90 sec.	5	4	9
Total	5	15	20

(4) We compute the probability of getting five out of 20 brittle bars on the assumption that the probability of a brittle bar is p'. $\left(= \frac{20!}{5!5!}\, p'^5(1-p')^{15}\right)$

(5) We take the ratio of (3) to (4) to get the conditional probability of getting the split obtained, given the marginal totals. $\left(= \frac{9!5!5!}{5!4!20!} = .0081\right)$

Thus at the 0.05 level, both χ^2 and the exact method would lead to rejection of the hypothesis of independence (i.e., we would conclude that heat treatment does affect brittleness.) At the 0.01 level, however, the corrected χ^2 would not lead to rejection while the exact method would. For small samples of 40 or less Mainland's tables of significant differences in fourfold contingency tables[9] will be found very helpful.

THE ADEQUACY OF χ^2 TESTS

The results of the tire problem are a little surprising since our general experience regarding the manner in which we park a car suggests that the right rear tires are likely to wear more relative to right front tires, than left rear tires do relative to left front tires. The data themselves are in line with this expectation, but the two sample percentages, 43 percent for right rear as against 36 percent for left rear, are not sufficiently different to be significant. This raises the question of the adequacy of our test. Are 400 tires enough to detect the difference that might actually be there? Now, it should be recognized that if enough tires are taken it would be possible to detect any difference that existed, however small. The practical question is not whether the sample is large enough to detect *any* difference, but whether it is sufficient to detect a difference of *importance*.

The ability of a test to detect differences is given by the OC curve for the test. For χ^2 tests of independence departures from the condition of independence are measured by

$$\lambda = \Sigma\left[\frac{(m_i' - m_i)^2}{m_i}\right] \qquad (3)$$

where m_i is the frequency of the i^{th} cell that is expected on the assumption of independence and m_i' is the frequency on some alternative basis. Both the m_i and m_i' must, of course, sum to the marginal totals given in the problem. For this measure of departure from independence, Miss Evelyn Fix has tabulated abscissa and ordinates for OC curves for degrees of freedom from one to 100. A selected set of her results are presented in Table VII.

Let us determine now the 0.10 point of the OC curve for the tire test run above. In any 2 x 2 table such as Table V, designated differences between m_i' and m_i for the given cells must be the same for all cells, since as noted above m_i' and m_i must sum to the same row and column totals (i.e. the $m_i' - m_i$ must sum to zero by rows

TABLE VII—Values of λ for which the Probability of Falsely Accepting the Hypothesis of Independence is 0.10*

(Probability of Falsely Rejecting Hypothesis of Independence $= \alpha$)

Degrees of Freedom	λ	
	$\alpha = 0.05$	$\alpha = 0.01$
1	10.509	14.879
2	12.655	17.427
3	14.172	19.248
4	15.405	20.737
5	16.470	22.033
6	17.419	23.187
7	18.284	24.238
8	19.083	25.211
9	19.829	26.122
10	20.532	26.981
20	26.132	33.852
30	30.379	39.074
40	33.941	43.461
50	37.069	47.312
70	42.48	53.99
100	49.29	62.39

*Abridged from Reference (7). Smaller λ's yield higher probabilities of accepting the hypothesis of independence.

and columns.) Call this common difference d. Then for a 2 x 2 table we have

$$\lambda = d^2 \Sigma \left(\frac{1}{m_i}\right) \tag{3a}$$

To find the d (for the given m_i) that has a 0.90 chance of being detected we set $d^2 \Sigma(1/m_i)$ equal to the $\lambda_{.10}$ for one degree of freedom. If we take a significance level of 0 05, this gives us

$$10.509 = d^2 \Sigma \left(\frac{1}{m_i}\right)$$

For the data of Table V, which gives the m_i for the tire problem, we have

$$10.509 = d^2 \left[\frac{1}{108} + \frac{1}{132} + \frac{1}{72} + \frac{1}{88}\right]$$

from which

$$d^2 = \frac{10.509}{.042088} = 250$$

and $d = 15.8 \simeq 16$. Hence, if the true expectations (the m_i') were as shown in Table VIII (note that $m_i' = m_i \pm d$), there would still be a chance of 0.10 of not detecting such a situation.

In terms of percentages, Table VIII means that in reality the percentages could be 47 percent for right rear and 31 percent for left rear, a difference of 16 percent, and we would still have a chance of 0.10 of saying that there was no difference. If we should feel that a difference of ten percentage points, say, was important enough to detect, then the above test is definitely not adequate since the risk of failing to detect a difference of ten percentage points is greater than 0.10.

What total sample size then *would* be adequate to detect a difference of 10 percentage points? Fortunately the analysis of the preceding paragraph can be put in general terms. Thus, if we wish a 0.90 chance of detecting a difference of δ percentage points (expressed as a fraction, say, 0.10), if we agree to take equal sample sizes (i.e., $n_1 = n_2$), and if we estimate that the average percentage of the class in question is p_o, then the total sample size to take is

$$n = \frac{42.036}{\delta^2} p_o (1 - p_o) \tag{4}$$

where $n = n_1 - n_2$. Thus suppose we agree to examine an equal number of left and right tires, and let us estimate the average percentage of rear tires scrapped (on the left and right sides combined) as 40 percent. Then, to have a 0.90 chance of detecting a difference of ten percentage points, we should take $n = \frac{42.036}{(.10)^2}(.4)(.6) \simeq$ 1000 or 500 left and 500 right tires.

If for some reason or other the samples are to be split in other than a 50-50 ratio, say one sample of rn and another of $(1 - r)$ n, then the formula to use is

$$n = \frac{10.509\left(\frac{1}{r} + \frac{1}{1-r}\right)^2}{\delta^2\left(\frac{1}{rp_o} + \frac{1}{r(1-p_o)} + \frac{1}{(1-r)p_o} + \frac{1}{(1-r)(1-p_o)}\right)} \tag{5}$$

It is to be noted that the above formulas are not valid if they should yield very small n's. This, however, is rarely the case for the values of δ that we are usually interested in.

When we are concerned with an r x c contingency table for which r or c or both are greater than two, formulas (3a) and (4) do not apply and we must go back to formula (3). In this instance simplification in terms of some meaningful quantity may not be possible and the best we can probably do is to try out various values of m_i' for given m_i and note how close the λ's come to $\lambda_{.10}$.

LIMITATIONS ON SIZE OF EXPECTED FREQUENCIES

A final example will illustrate certain other aspects of χ^2 tests. The data for this example are shown in Table IX. They pertain to accidents reported in 1951 for certain makes of single-engine aircraft classified by type of accident. The question is whether the planes differ significantly in their accident "profiles."

The first thing that will be noticed is that there is a large number of categories and in some a rather small number of cases. Since the use of probability values taken from an ordinary χ^2 table involves some approximation, as noted above, it is necessary in running a χ^2 test of independence to be on our guard for very small expected values.

The rule followed for many years was that no cell should have an expected frequency less than ten (some said five) cases. More recently the rules have been considerably relaxed. W. G. Cochran[1] lists the following recommendations:

> (a) The 2 x 2 table. Use Fisher's exact test (i) if $n < 20$, (ii) if $20 < n < 40$ and the smallest expectation is less than five. Mainland's tables . . . are helpful in all such cases. If $n > 40$, use χ^2, corrected for continuity if the smallest expectation is less than 500.

TABLE VIII—True Expectations That Would Have a 0.90 Chance of Being Detected by a χ^2 Test Applied to Data Whose Marginal Totals Are the Same as Table IV.

	Front	Rear	Total
Left	124	56	180
Right	116	104	220
Total	240	160	400

TABLE IX—Accidents of Three Makes of Single Engine Aircraft in 1951*

(Frequencies Expected on Hypothesis of Independence Shown in Parentheses)

Type of Accident	N–	B–	T–	Totals
Ground & Water Loop	2 (1.9)	2 (2.2)	3 (2.8)	7
Hard Landing	4 (4.8)	2 (5.8)	12 (7.3)	18
Undershoot	2 (3.5)	5 (4.2)	6 (5.3)	13
Overshoot	7 (3.2)	3 (3.9)	2 (4.8)	12
Nose Up and Over	3 (4.6)	0 (5.6)	14 (6.9)	17
Collision:				
Between 2 in Air	0 (2.47)	3 (2.9)	6 (3.6)	9
With Ground or Water	11 (7.8)	8 (9.5)	10 (12.1)	29
Other	22 (23.2)	23 (28.1)	41 (34.8)	86
Stalls	7 (14.8)	12 (18.0)	36 (22.2)	55
Air-Frame in Air	3 (2.7)	7 (3.2)	0 (4.0)	10
Air-Frame on Ground	17 (8.3)	11 (10.1)	3 (12.5)	31
Other	12 (12.7)	33 (15.7)	2 (19.0)	47
Totals	90	109	135	334

*From Civil Aeronautics Board.

(b) Tables with degrees of freedom between two and 60 and all expectations less than five. If n is so small that Fisher's exact test can be computed without excessive labor, use this. Otherwise use χ^2, considering whether this needs correction for continuity by finding the next largest value of χ^2.

(c) Tables with degrees of freedom greater than 60 and all expectations less than five. Try to obtain the exact mean and variance of χ^2 and use the normal approximation to the exact distribution.

(d) Tables with more than one degree of freedom and some expectations greater than five. Use χ^2 without correction for continuity.

The data of Table IX have only one expected value less than two and many greater than five, so we shall apply rule (d) and use the straight χ^2 formula without correction for continuity. The value of χ^2 is 108.9. This is far beyond the $\chi^2_{.05}$ point, indicating a significant difference among the planes.

INDIVIDUAL COMPARISONS IN AN R × C TABLE

Having noted that the planes of Table IX differ significantly, we may find it of interest to compare the planes with respect to individual accident categories. This can be done by setting up 2 x 2 tables in which a particular accident category serves as one class and all other types as the other class. Two particular planes can make up the other classification or one plane can be set against the other two as a group. For example, to compare planes N– and B– with respect to stalls, we have the table:

	N–	B–	Total
Stalls	7	12	19
All others	83	97	180
Total	90	109	199

For this, $\chi^2 = 0.62$ which is less than the $\chi^2_{.05}$ point for one d.f. We can now compare N– and B– as a group with T–. This yields

	N– + B–	T–	Total
Stalls	19	36	55
All others	180	99	279
Total	199	135	334

Here $\chi^2 = 17.3$, which is beyond $\chi^2_{.05}$ for 1 d.f. and indicates that plane T– has relatively more stalls than planes N– and B–.

It also may be of interest to compare individual planes over all accident categories. Thus we can run a χ^2 test on N– and B– accidents alone. If this is done we get a χ^2 of 22.6 which is greater than $\chi^2_{.05}$ for 11 degrees of freedom, indicating that planes N– and B– differ significantly. More detailed analysis indicates that this significant difference arises mostly from the difference in the relative number of accidents falling in the "other" category. Similar comparisons between N– and T– and B– and T– also yield significant χ^2, but in this case the large differences appear in such categories as air-frame ground accidents, stalls, and overshoots as well as in the "other" category, indicating a more striking difference between T– and other planes.

This accident analysis raises some nice questions about interpretation of χ^2 tests of independence. The analysis shows, for example, that among the accidents of plane T– there is a significantly greater percentage of stalls than among the accidents of planes N– and B– as a group. Thus if we knew we were going to have an accident and were in plane T–, the probability of it being a stall would be higher than if we were in planes N– and B–. There is nothing in the analysis, however, that tells us anything about the relative probabilities of an accident in the various planes. The total accidents for each plane depend as much upon use as upon accident propensity. In the absence of supplementary data on flying time, therefore, it is not possible to conclude that from the point of view of stalls, T– is a riskier plane than N– or B–. This could be concluded only if all planes had been flown the same number of hours under similar conditions.

There is, therefore, no magic in χ^2 tests that allow us to depart from the basic principles of experimental design. If there is interest in certain conclusions, the conditions of the experiment should be such that these conclusions can either be accepted or rejected.

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The Challenge of SQC in Manufacturing Plastics Raw Materials*

H. L. WEHRLY

The Dow Chemical Company, Midland, Mich.

In the manufacture of plastics raw materials, or manufacture of chemicals for that matter, there are certain traditional quality controls which have been built-in as a result of the normal development of the product involved. The established and routine methods of statistical quality control, such as the various control charts, simple correlation, and the other techniques which have been standardized to a great extent and applied to the machine tool industry, automotive industry, etc., have not always been found to be applicable, as developed, to certain processes in chemical manufacture. So, when SQC was first presented to our management they argued that, fundamentally, the same techniques were already being used, and on a 100 percent basis. SQC was faced with answering the question, "Why should we use a method which normally requires sampling and partial inspection when we are automatically using 100 percent inspection?"

NORMAL SQC NOT APPLICABLE TO CHEMICALS

To demonstrate what we faced in trying to sell SQC, we might go through some of the functions of modern chemical or plastics manufacture. It should be made clear that when we say plastics manufacture we are not referring to the large molding industry, but to the basic chemical process of making the raw molding materials.

The design of production processes in the chemical industry has to start with basic chemistry. The processes, and process equipment are then developed step by step through the sequence of laboratory, semi-plant, pilot plant, and then on to the production plant. The development of these designs is unique. The design of these processes, like those of a special tool design, have to be set up with a specific and special end use in mind.

As an example, let's follow the development of a typical process which today produces millions and millions of pounds per month. Let's take the process of polymerization of styrene monomer. In the early stages of the development of polystyrene, the chemists in the laboratories found that its polymerization was a normal and simple chemical reaction. They soon learned how to control the polymerization in small laboratory equipment. They made enough of it so that preliminary basic tests could be made to determine if the product would have any acceptance in general trade. Once that was determined, the problem then was the development of large scale production facilities to produce a uniform product with good characteristics. The processes resulting from the laboratory stages consisted of completely custom built equipment in both pilot and production plants in almost 100 percent of the cases. This custom built equipment had to be duplicated in many cases in order to achieve the poundage necessary, so automatic controls and complete in-built quality controls had to be incorporated as a part of the normal process. These controls in effect take the place of the quality control inspector with his sampling and control charts. These processes are controlled with recording devices which control the pressures and temperatures and thru-put feed to very close tolerances. So, in effect, the trained operator is actually the quality control operator. His control charts are the charts of the automatically recording instruments. His job is to make process adjustments when his instruments tell him to do so. If he does his job correctly the product emerging from his process will be as designed and of the proper quality.

Regardless of the accuracy of the operators and instrumentation in the process control, the product must be inspected for characteristics which cannot possibly be determined by the process controls. This inspection is, in almost all cases, carried out by the control laboratory. They sample the flow of the product whether it be batch or continuous, and analyze the samples for conformance to physical property characteristics. The techniques of sampling and analysis of either batch processes or continuous processes as developed in the chemical industry are looked upon as 100 percent inspection. Generally, it is felt that this type of inspection is necessary in this type of business, and management generally looks upon a typical SQC sampling plan with some degree of skepticism.

In the chemical industry most production plants are, in a sense, considered as pilot plants, in that major changes in the production process are continually being made toward a better product. Process engineers are constantly striving to use better instrumentation, more automatic controls where possible, and better process design; all aimed at a better end product.

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PLANT SHUTDOWN FOR CORRECTIVE ACTION TOO COSTLY

There are other difficulties in fitting normal SQC techniques into chemical operations. The standard SQC control chart, for instance, will forewarn the operator of, say a milling machine, of impending trouble. The procedure then is to shut the machine down, make the corrections, and then proceed with production. This usually takes just a matter of minutes or at the most, hours. In the chemical industry, the shutdown of a process may require not minutes or hours, but in some cases, days. The costs of shutting down batch-wise processes or continuous processes is tremendous, and management insists upon much closer in-process control than might normally be insisted upon in industries where shutdowns are not so time consuming and costly.

Another area of difficulty in installing SQC in the chemical industry generally, is in incoming inspection of certain raw materials used in the processes. Normal inspection of raw materials as units is rather impossible because of the fact that the raw materials are not readily identifiable as units since they come from a variety of tanks and pipe lines and are supplied in continuous flow.

To summarize why SQC is difficult to sell in the chemical industry, we might say simply this: The basic principles of SQC are already being used in that they have, of necessity, been built into the processes as they have been developed.

SQC TECHNIQUES CAN BE APPLIED

Now, let's look into the brighter side of the picture from the standpoint of statistical quality control.

In spite of all the objections to SQC just stated, there are areas of operation where the methods can be used without objection—areas where they are actually needed.

It was in these areas that we were able to get a foothold, so to speak. What did we do? We had a management that was willing to accept statistical quality control techniques and had given its authorization to install them if we could prove our case. We were also given an adequate budget which was necessary to do the proving. There is where the challenge entered. What was the best way to meet that challenge! We felt that the idea of first laying out a complete, well-rounded plan of operation for statistical methods, and then introducing that plan, was not a good one. Rather than that, we worked on that rather well established theory that before statistical quality control can be successful, we must have quality mindedness. Statistical quality control is but one tool in a good quality program. We, therefore, set up over-all quality programs which resulted in creating a quality mindedness in all personnel from janitors to managers. To go into details of such a program is a subject in itself. In our case, the elaborateness of the program depended upon the size of the department in some cases, or upon managements' willingness to pay in others. We found that once the ground work of quality mindedness was laid, the applications of statistical quality control, as a tool in itself, became much easier.

Figure 2—Summary Charts for Process Controls

Let's assume that the programs for quality mindedness had their effect and that we now do have a general level of quality mindedness among the personnel in production and laboratories. Where do we apply the first statistical quality control, and what statistical quality control technique do we use? In any production operation, it is easy to pick a spot where a simple control chart can be installed. Through that you can gain the confidence of a great many people in the department. We used the $\bar{X}$, R charts on filled package weights. This proved to be a happy choice since we could almost immediately show management some tangible savings in costs. The savings here resulted from cutting over-weight which was being shipped to the customer.

Another area of operation for statistical quality control which is rather easy to put into effect is in the inspection of incoming raw material. The reference here is not to the raw materials mentioned before, but to such things as empty packages, drums, and additives purchased as units. This inspection based on good statistical technique results in more complete specifications, and therefore better raw materials. Our use of the $\bar{X}$, R chart in one department gained the confidence in SQC that we needed among the people in that department. We were then in a position to begin use of our general

Figure 1—Chemical Process Control Charts

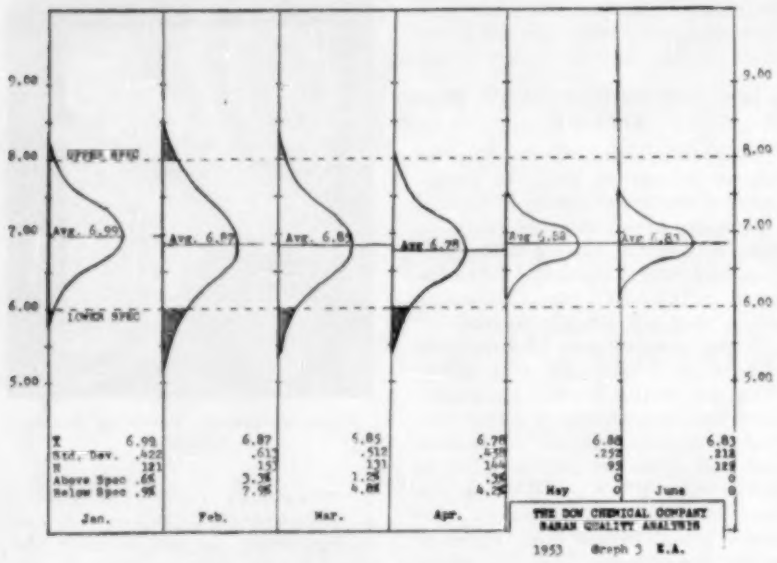

Figure 3—Quality Report—Saran Quality Analysis

plan of operations in statistical methods.

Aside from package weights and incoming inspection, how else can we use statistical quality control in the manufacture of chemicals or plastics raw materials? There are a great many ways and a great many techniques, some of which you won't find in the text books. Some are new-born methods which may or may not involve any statistical manipulations. Some of the ideas we use do not involve any formulas or mathematical techniques at all, they are merely off-shoots of things we have learned through statistical quality control.

SQC METHODS IN THE LABORATORY

One of the first areas of operation for statistical techniques is in the laboratory. We have found that the data gained from laboratory analysis of materials is only as good as its accuracy, and that is not unique with us. We have also found that inaccurate data from the laboratory invariably tends to destroy the confidence of production people who depend on these data for release of material for shipment. For example, this sort of thing leads to the sometimes-heard-of practice of shipment of material by production management regardless of quality level reported by the testing or inspection groups. We have found that proven accuracy in laboratory data tends to dispel any lack of confidence production has in the data it uses for release of materials for shipment. We are not going into the details of how the lab accuracy has been improved, but various statistically designed experiments were used in establishing a level of laboratory accuracy which left no doubt of its accuracy in the minds of the people who depended on it. In other words, the laboratory which is under constant statistical control is going to produce results which will be a great potential for building goodwill between production people and laboratory or inspection departments.

In our plant, because of the studies we made of laboratory accuracy, we have advanced that technique into other areas of operation. For instance, we have established a routine pattern of inspection of such things as the automatic recording machines and the automatic recorders in process. We know that the process is only going to be as good as the instrumentation and if the instrumentation is out of repair, the results are disastrous. We also have developed a routine pattern of examination of certain key points in processes, such as filters or metallic eliminators, to check them for their condition and efficiency. Such things as meters and scales are also checked for accurate measurement in a routine manner. All of these things have been developed as a result of a basic use of statistical methods.

USE OF CHARTS AND GRAPHS

One of the most unique and important tools we have used involves a system of charts containing data that we get from the laboratory analysis of the various characteristics of a product. Figures 1 and 2 are shown to furnish an idea of the size of these charts.

These charts are based on accurate laboratory and inspection data. They are used for a continuing coordination of effort between operators, foremen, and superintendents. Weekly meetings are held around these charts and the problems of production are discussed among the people present. They have been a very good factor in making the operators, especially, feel that they are on the team.

Another form of report (see Fig. 3) is designed for management. It makes use of simple distribution curves and gives management at a glance a picture of how they stand with their product. It also gives them comparative figures from month to month and in some cases, we have charts showing the data from year to year. These charts eliminate the necessity of going through a whole list of reports or listed data to get information that is needed. It's there at a glance.

We have applied our statistical methods in certain areas of studying quality deviation reports, which in a different language means simply complaints. Good, sound quality deviation analysis results unquestionably in bringing to the attention of the proper people the necessity for improved quality and in getting the action necessary to obtain it. In some cases these reports bring out the possibility of readjusting certain specifications *where too much* quality is present. Figure 4 is an example of the type of line chart we use for reporting to management.

To a certain degree statistical quality control has met many challenges in our operations. Production

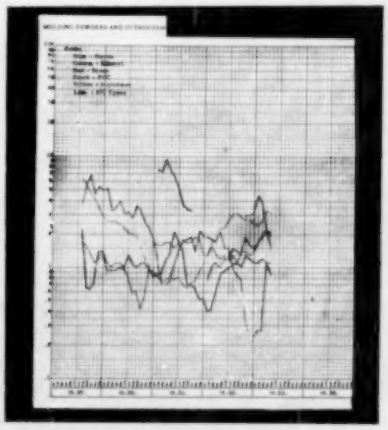

Figure 4—Management Report on Molding Powder and Styrofoam Quality

problems come up sometimes which bring about disagreement between production and research labs and process engineers. We have found that when a quality control engineer has been called in on such problems, he has been able, by the use of certain SQC methods, to solve some of the problems or at least get them on the way toward solution. In problems of this sort, it is more and more evident that a good quality control engineer, or a good quality control consultant, can be extremely valuable in bringing about good coordinating efforts between production control and research and process people. We have established the practice of having an SQC consultant available on a regular schedule, to assist with and coordinate the work of the regular quality control engineers. His job is to inject fresh and unbiased ideas into quality control groups.

The challenge which any production operation can give to statistical control are limitless. We have only high-lighted some of the challenges which we have taken on, attempting merely to show where, to a greater or lesser degree, we have made progress. We are by no means completely organized; actually we have only scratched the surface. Because of the experience we have had so far with the use of statistical methods, our long range plan of operation on an over-all basis very definitely includes any and all statistical quality control tools that will be of value, and there are many of those tools.

Acceptance Sampling

Another Method of Clerical Quality Improvement

DAVID J. BLACKWELL
Prudential Insurance Company of America, Newark, N. J.

Acceptance sampling is the most recent addition to the list of techniques which we have borrowed from statistical quality control for use in our Quality Improvement Programs in the Prudential. In the past, we have made use of statistical sampling and analysis with marked success in helping to reduce clerical errors. The fraction defective control chart has been particularly useful in determining the significance of changes in our sample results. We have found less application for the control chart for variables, since clerical work does not lend itself easily to measurement by variables.

Each of our individual Quality Improvement Programs for work such as correspondence, calculating, filing, keypunching, card picking, typing, posting of information, and complex case preparation has been tailor-made. Before the use of Acceptance Sampling, each program had had one or more of the following accomplishments as its primary goal.

1. *The elimination of systematic errors*

   Any clerical operations may have systematic errors caused by lack of education, conflicting rules, or faulty forms. Often these errors go unrecognized even when work is checked. Analysis of sample error experience will usually discover this type of mistake.

2. *Standardization of procedures*

   Although this field is closely akin to the elimination of systematic errors, there is a peculiar trait of clerical work which makes its separate consideration important. Often there are several different ways of completing a clerical operation satisfactorily. Since many of our clerical operations are interrupted for additional information, completion by a second person can be delayed and confused by lack of standard procedures of preparation.

   The Quality Reviewer, or inspector, is usually a member of the section being reviewed. The job as Quality Reviewer is rotated among the clerks of the section. Thus, our Quality Reviewers are well equipped to recognize and suggest corrections of non-uniform procedures.

3. *Education of clerks*

   The clerks are educated through the errors found in the Quality Review. The Quality Reviewer returns each error found to the section supervisor who in turn discusses the case with the clerk who has made the error. These discussions are aimed towards preventing future errors rather than at correcting the present error itself. The clerks know that no records are kept of the individual's error performance. Therefore, they tend to welcome the instruction, since it can prevent future errors which might be embarrassing if detected by an outside source.

   The Quality Reviewers, themselves, are educated by being given the opportunity to study their jobs without the pressure of normal production.

4. *Reduction of accidental errors*

   When careless, or accidental errors, are brought to the attention of the clerks, and they realize that nothing systematic has caused the mistakes, the natural human tendency to try to improve makes them examine their own work habits. The improvement of these habits will usually reduce accidental errors.

5. *Complete elimination of uneconomic checks*

   Often, when clerical errors in first work have been reduced, checks or second checks become economically unsound. At times, first or second checks are unsound before any reduction in errors. In either case, the Quality Program may be designed to measure the breakeven point in accuracy and completely eliminate checks which are not paying for themselves. Since these checks are eliminated only when the accuracy is consistently well above the breakeven level, their elimination has none of the elements of acceptance sampling. This is true, I believe, even though we use the results of continuing Quality Review to assure ourselves that the accuracy level has not changed.

CLERICAL ACCEPTANCE SAMPLING

Recently, we found an application for Acceptance Sampling which fitted the basic requirements of any such plan:

1. Large volume of work units
2. Comparable work units
3. Work units classifiable as correct or incorrect
4. Work units retainable until decision reached to accept or reject

THE PROBLEM

A central records unit, known as the Register Division, maintained two types of records which were similar and, to a certain extent, overlapping.

The first of these records was the Register Books, ledgers with consecutive entries for each Ordinary life insurance policy issued from the beginning of our company in 1875 to a later year when a more modern method of recording was established. These Register Books contained an entry for each of the first 10,200,000 Ordinary policies issued by the Prudential. Naturally, the vast majority of these policies were no longer active, because of normal deaths, lapses, and surrenders over the years. Of the total 10,200,000 policies issued in this group, approximately 1,750,000 were still on an active basis.

For these 1,750,000 policies, a second record, the Renewal Card, was maintained. It was an index card type of record. Many of the items of information which appeared in the Register Books were also shown on the corresponding Renewal Cards. Maintenance of these common items of information caused considerable duplication of effort. The Renewal Cards also were used to maintain a history of premium payments until the premium accounting was converted to an I.B.M. procedure. The elimination of the double record was one of the major justifications for the conversion.

When this conversion was completed all of the necessary information could be kept conveniently on a single record card. It was also realized that the cumbersome handling of the Register Books, each of which contained information for approximately 2,000 policies, could be avoided.

Various methods of combination were investigated. It was finally decided that the transcription of the few non-common items of information from the Register Book to the Renewal Cards could be accomplished most economically by a manual transcription.

PREVIOUS EXPERIENCE

This same method had been used for a similar transcription when the records for one of our newly organized Regional Home Offices were being segregated. That transcription project had had a 100 per cent inspection.

The original plans for the transcription of the 1,750,000 cards also called for a 100 per cent inspection. Those of us in the Quality Improvement Staff felt that three things should be true of the transcription.

1. No 100 per cent inspection would eliminate all of the first work errors.
2. Some of the individual first workers would produce work which would be so accurate that its 100 per cent inspection would be economically unsound.
3. Use of error experience as an education tool could improve the accuracy of first work to a point where most of the unchecked first work would be better than the 100 per cent checked work from the previous job.

For these reasons, a random sample of the previous transcription was made. This sample indicated the percentage of error in the finished product, i. e., work 100 per cent inspected. Although the amount of error found was not considered economically dangerous, we felt sure that an Acceptance Sampling program could produce a more accurate finished product with a considerable savings in inspection expense.

PROGRAM DESIGN

With these thoughts in mind we designed an Acceptance Sampling program for the project. We knew that the project would require 80 clerks for approximately 25 weeks.

A. Lot Size

We decided to consider each clerk's work for a week as a unit for acceptance or inspection. The work could be held conveniently for that period of time without disrupting normal maintenance of the records. This large lot size also would make a large sample practical.

B. Sample Procedure

Because of the physical characteristics of the Register Books, it was necessary to have the clerks transcribe all of the Renewal Cards corresponding to a book as a group. The number of individual transcriptions involved varied considerably from book to book, even though each book contained 2,000 policy numbers. In the earlier years of issue as few as 20 of the 2,000 were still active; in the later years, as many as 750.

For this reason, it was not practical to establish a set sample size per Book. Furthermore, it was essential to sample each Book and the corresponding cards as they were completed. Acceptance or rejection of each clerk's work for the week had to be decided quickly at the end of the week to determine the amount of staff which would be required for week-end overtime 100 per cent inspection.

We also wanted to return the individual errors to the clerks through supervision as soon as possible after the Book had been completed. This could give our training program maximum effectiveness.

We settled on a ten per cent single sample. This sample was taken using one number in the terminal digit position. The number was changed daily and was not known by the transcription clerks.

C. Quality Review Staff

Whereas the clerks doing the transcription were new in the company, the people chosen to do the sample reviewing were clerks very familiar with both of the records. Our Quality Review Staff consisted of six reviewers and one supervisor. Their sampling was resampled to guarantee the successful performance of our Acceptance Sampling procedure.

D. Sampling Plan

Since practical considerations had forced us to take a flat ten per cent sample of the work completed, it became obvious that we would not be able to use either the Dodge-Romig Single-Sampling AOQL Tables or

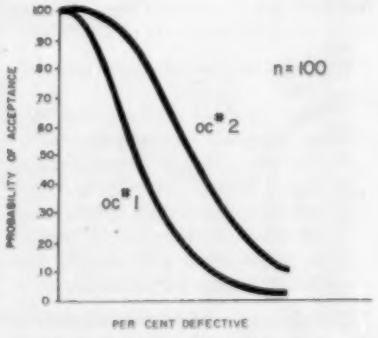

Figure 1—Transcription Acceptance Operating Characteristic.

the Military Standard tables. We were forced to design our own plan.

Our first step was to establish an AOQL, (Average Outgoing Quality Limit) or maximum AOQ, (Average Outgoing Quality). Although we had the error experience from the previous transcription to aim for, we deliberately set our standards much higher. We did this for several reasons:

1. This was our first attempt to use Acceptance Sampling and we were anxious to produce rather dramatic results.
2. We knew that our inspectors, or Quality Reviewers, would make errors themselves because of the routine monotony of their job.
3. We felt that the transcription clerks could produce work much better than the previous transcription if proper training and error follow-up were used.

For these reasons we selected an AOQL which was approximately half of the error rate found in the previous job. To allow for the wide range of weekly production from individuals we used the plans:

N=590 (approx)	N=1,000 (approx)
n= 59	n= 100
c= 1	c= 2

N=1,400 (approx)
n= 140
c= 3

Any lot for which the sample was less than 59 was automatically 100 per cent inspected.

Actually these plans should give us something better than the AOQ selected for two reasons:

1. Defectives found in the sample of lots which will be accepted are eliminated.
2. We could not stop sampling when n = 100, for example, because we never knew when another group of work would need to be added to the individual's work-week lot. Since our errors were bound to occur in integral numbers, an average sample size of 120 obviously would have given us better work than our expected AOQ if "n" theoretically equals 100.

These advantages were in part offset by the expected Quality Reviewer error.

The operating characteristic curve for n = 100 is shown in Fig. 1 (OC#1). OC#2 which also appears in Fig. 1 represents a plan which was used later in the project. It is discussed later in connection with our experience during the project.

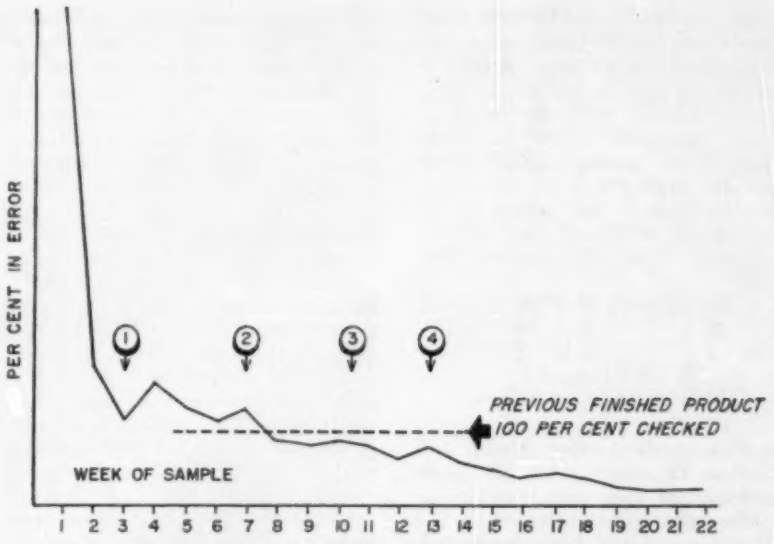
PER CENT IN ERROR
1
2
3
4
PREVIOUS FINISHED PRODUCT
100 PER CENT CHECKED
WEEK OF SAMPLE
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22

Figure 2—Quality Index Home Office Transcription Unchecked Work.

RESULTS

Our experience during the actual operation of the project was very satisfactory. We received the utmost cooperation and assistance from management, supervision, and the planning group who were all responsible for the transcription.

From the very beginning of the project, the training program designed by the planning group and our reference of errors provided much more effective education of clerks than had been experienced in the previous transcription. Figure 2 indicates that the transcription was highly defective during the first week of the project. This was not at all surprising, since the clerks were starting their first job in the company.

The process average showed a continuous improvement with the exception of points (1), and (2) and (4) indicated in Fig. 2. In each of these cases there was an assignable cause responsible for the departure from the continuous improvement.

The apparent improvement indicated by arrow (1) was fictitious and was caused by a degree of laxity in our Quality Reviewers. This situation was recognized in our resampling of the Reviewer's sample and was corrected. Points (2) and (4) indicate weeks when large groups of new clerks were added to the project. Because training techniques had been greatly improved, these effects were short-lived.

Point (3) indicates the time when the one major change was made in the program. Those who were responsible for the project wanted to

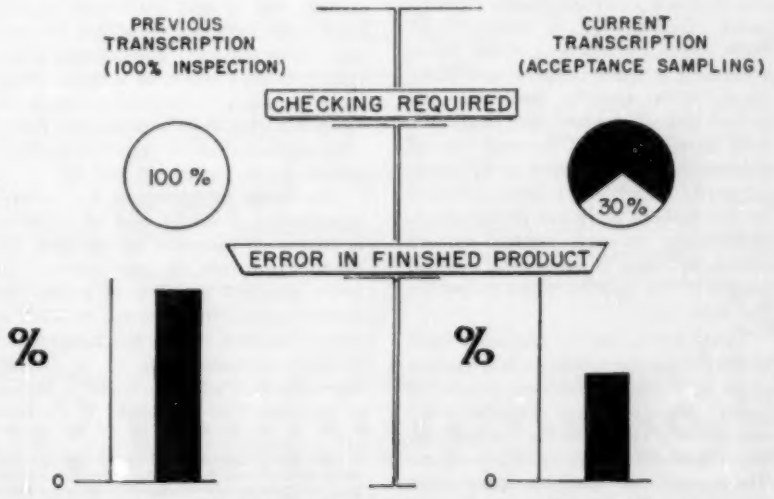

Figure 3—Manual Transcription Error Experience and Checking Requirements.

obtain the maximum benefits from Acceptance Sampling commensurate with reasonable accuracy. After the process average (as indicated by the weekly sample of 10,000 cards or more) had been below the error found in the finished product of the previous transcription for three consecutive weeks, they asked us to change our sampling plan. A new maximum AOQ was selected. The plans

N = 360 (approx)	N = 560 (approx)	
n = 36	n = 56	
c = 1	c = 2	
	N = 800 (approx)	
	n = 80	
	c = 3	

fit this standard very closely, and the last 12 weeks of the 22-week project used these new plans.

When the transcription of the 1,750,000 cards had been completed, we took a large random sample of the completed job. This sample included work which had been accepted and work which had been rejected. We found one very interesting thing which bore out one of the assumptions we had made before the project started. The work in the finished product which had been rejected and consequently 100 percent inspected still had an error rate greater than that work in the finished product which had been accepted on the basis of the sample.

Through the Acceptance Sampling Program, we were able to avoid 100 percent inspection on 1,213,180 of the 1,756,914 cards transcribed. (70 percent of the project). Included in the 30 percent which had to be inspected were all of the cards transcribed during the first three weeks of the project, the training batches from clerks who were added to the project, and the lots which were not large enough to have had a sample of 59 cards taken from them.

The total cost of the transcription project was approximately $115,000. Our Acceptance Sampling Program saved $25,000 by eliminating the checking operation on 70 percent of the cards, and the finished product was 43 percent more accurate than the goal we had set for ourselves.

All of these accomplishments were possible only through the conscientious efforts of everyone concerned with the project; the transcription clerks, the Quality Reviewers, supervision, planning, and management. This is true of all clerical quality improvement programs since we are dealing only with people, and not with machines or materials.

Thus Acceptance Sampling has become an integral part of the techniques we are using to improve quality at the Prudential. We are ready to adapt it to other clerical operations, with the knowledge that it may help us attain our general Quality Improvement goal—more accurate work at a similar or smaller cost.

Simplifying Quality Control Operations with Marginal Punch Cards

BERNARD GIFFLER*

DESCRIPTION

Since this article deals with the uses of marginal punch cards, it might be profitable at the outset to make sure that everyone knows what these cards are. Figures 1 and 2 illustrate these cards.

As can be noted, these cards are pre-punched about their edges with one or more rows of equally-spaced holes. Information is enclosed in these cards by notching these holes according to some prearranged plan. Cards with specific messages are sorted out of a stack by passing a steel needle (Fig. 1) through one of the holes in such a way as to cause all cards which have been notched on this hole to drop out of the stack. Depending on the coding system which is used, it may take several passes of the needle to get a specific message.

These cards can be obtained from several manufacturers[1] in a variety of sizes, pre-punched, and printed to order. Most of these manufacturers also sell such items as sorting needles, hand punches, correction paper, etc. They are more likely to lease more expensive items as key and gang punches. The author has found the charges for this equipment to be quite reasonable.

SELECTIVE CODING SYSTEMS

Codes for marginal punch cards are of two basic types: selective and selective-in-sequence. The selective codes are those which permit the sorting out of any individual message with just one needling operation. The simplest case of selective coding occurs when, in a single row of holes, each individual message is assigned its own individual hole. This special type of selective coding is also known as direct coding.

The basic requirement for selective-coding a single row of n holes is that each message be encoded by notching a unique combination of some constant number of holes. In consequence, the capacity of such a row of holes is simply the number of possible combinations of x things taken from n where x is the number of notches per message. The word "field" is frequently used to refer to any row of holes (or rows of holes, if there are more than one) in which an individual message out of some set of alternative messages may be encoded. When selective coding is used with two or more rows of holes, it is necessary that fewer notches be used per message in each successive row in from the edge of the card.

Since the capacity of a selective-coded single row of holes is merely the general term for the coefficient in the binominal expansion, it is obvious that maximum capacities will occur at $x=n/2$ (if n is even) or $x=(n\pm1)/2$ (if n is odd). The expressions for the maximum capacities for selective codes using two or more rows of holes are not difficult to obtain.

SELECTIVE-IN-SEQUENCE CODES

In contrast to the selective codes, selective-in-sequence codes may use any number of notches per message. The capacity of a field of n holes in r rows is simply $(1+r)^n$. The name "selective-in-sequence" arises from the fact that these codes may be sorted selectively (that is, with one

*The techniques described in this article were worked out during the time the author was employed by Alexander Smith, Inc., Yonkers, N. Y.

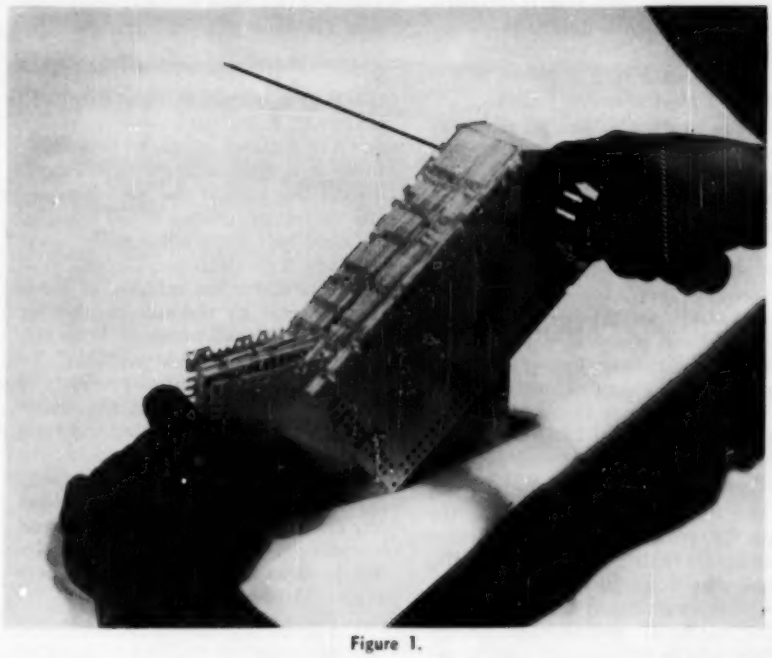

Figure 1.

needling operation per message), provided the messages are taken in a specific sequence.

Two very useful types of selective-in-sequence codes are referred to by the terms "binary" and "ternary." These codes permit the maximum utilization of a given message space and also provide special opportunities for reducing the sorting effort.

Binary codes are used with single rows of holes; ternary with double rows. As might be expected, these codes are based on the binary and ternary number systems; that is, systems using two and three repeating digits. The following table shows some arabic numbers and their binary and ternary equivalents.

Arabic	Binary	Ternary
1	1	1
2	10	2
9	1001	100
10	1010	101
15	1111	120
26	11010	222
32	100000	1012

From the table we see that all arabic numbers up to 15 can be restated as binary numbers in four or fewer digits. This being the case, we can encode these arabic numbers in a 4 × 1 field (that is, a field of four holes in one row) using one hole per digit. If the binary digit is one, we simply notch that digit's hole; if the binary digit is zero, we leave the hole unnotched. The actual translation of the arabic numbers into their binary equivalents is easily accomplished by using the series 1, 2, 4, 8, etc. as subscripts (that is, we have a different one of these numbers printed under each hole in the field) and by notching those holes whose subscripts add up to the arabic number which we wish to encode. If we have double rows of holes and are using ternary coding, the subscripts would be taken from the series 1, 3, 9, 27, etc. and would be repeated in each row.

GENERAL USE OF CARDS

Specific examples of the various types of coding to which we have referred are illustrated in Fig. 2 which is a carpet inspection card that is used at Alexander Smith. These cards are filled out by the carpet inspectors as they go about their work. They write in the grade numbers, looms and other required information in the spaces which are provided. They also indicate the actual defects they observe by notching (using for the purpose a small hand punch which they carry with them) the defects' code numbers which are listed along the bottom edge of the card.

All notching, except to indicate the code numbers of observed defects, is done in the quality control office. Here the work is greatly facilitated by using an inexpensive, hand-operated gang punch. At the end of the shift each inspector's cards are brought to the office and immediately gang-punched to record the date of inspection and the inspector's code number. Once this is done, the cards of several inspectors and shifts may

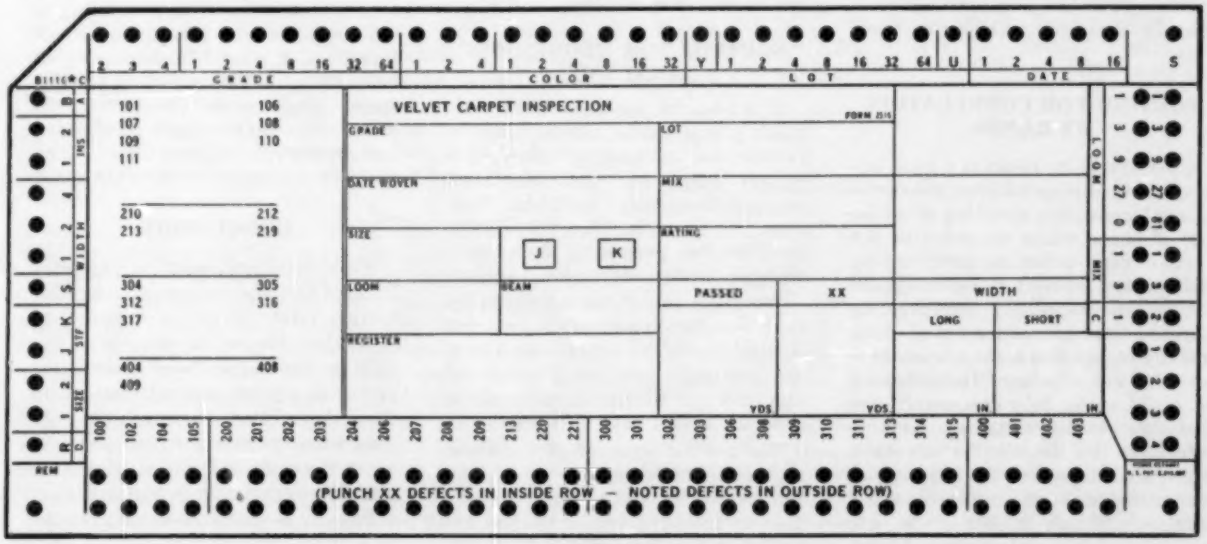

Figure 2.

be put together, visually sorted into groups containing identical messages, and then gang-punched. That information which it is not conducive to pre-sort and gang-punch is entered individually with an ordinary hand punch. Of course, the particular organization and equipment used at Alexander Smith may not be the most suitable in other situations.

The sequence sorting of a binary field is a relatively simple matter. These are the steps which must be followed:

1. Pass a steel needle through the 1-hole (the hole with the subscript "1"). Place the cards which drop out on this needling at the rear of the deck.
2. Repeat the previous operation in the 2-hole, this time being careful to keep the dropping cards in the same order which they occupied in the deck.
3. Repeat step 2 until each of the n holes in the field has been needled.

If the preceding operations have been correctly followed, the cards with no holes notched will be left on the needle while the deck, itself, will be in sequence, the "1" messages before the "2" messages, and so on. With n needling operations we can sort 2^n messages. Methods similar to the preceding can be used to sequence-sort ternary fields.

The sorting opportunities of these marginal punch cards greatly facilitates the presentation of the encoded data in all of their meaningful classifications. However, the usefulness of these cards does not end here; they can also be used to facilitate many formal statistical analyses which it may be desirable to apply to these data.

SORTING FOR CORRELATION BY RANKS

Correlation by ranks is a good example. As anyone who has ever tried it must know, the scanning of a long list of observations in order to put them in rank-order can become very tedious and subject to error. These errors are especially exasperating because one observation out of order usually means that quite a few others are also out of order. The difficulty in ranking the data can exceed the computational savings we hope to effect by the decision to use rank methods. However, the ranking of an extensive set of observations is a relatively simple matter if the data are entered in punch cards in binary or ternary code. All that is required in this case is an ordinary sequence sorting. The following example is similar to an actual problem which occurred at Alexander Smith.

It was desired to run an exploratory analysis by rank correlation methods of the relationships between certain carpet yarn characteristics and the appearance of the finished carpet. In all there were about 50 lots of yarn about which were known the average levels and standard deviations of the weight per unit length, turns of twist per inch, percent wool content, and the residual grease content after scouring. Corresponding to each lot of yarn, there was available the percentage of yards of finished carpet rejected for each of several appearance defects.

The first step was to simplify the data by reducing them to integral numbers ranging from zero to 100. This was done by discarding unnecessary significant figures, eliminating decimal points, etc. Since arabic numbers up to 100 can be restated as binary numbers in seven or fewer digits, we marked off fields of seven holes about the periphery of a blank punch card and assigned each yarn characteristic and percentage of rejected yardage to its own particular field. We then wrote each lot number on a separate card and notched into it all the data which pertained to that lot. When a card had been made out for each lot, we sequence-sorted the cards for a particular variate. This sequence sorting also put the cards in rank order with respect to that variate, which was our intention. In this example we sequence-sorted for each variate in turn and wrote the rank numbers on the card itself, directly under the field where the variate was encoded.

SORTING FOR REGRESSION ANALYSIS

It is easy to see how these marginal punch cards can be used to reduce the calculating effort in a formal regression analysis, using grouped-frequency methods. Suppose, for example, that we wish to analyze the preceding data as a multiple linear regression problem.

We begin exactly as we did in the rank correlation example except that we first divide the data for each variate into cells and enter these cell numbers into the punch cards rather than the actual data themselves.

The normal equations for a linear regression model require the calculation of the sums of the variables for each variate, the sums of the squared variables for the independent variates, and the sums of the cross-products for each pair of variates. The usual way to make these calculations is by constructing joint frequency tables as is shown in standard texts. However, if the data have been entered into punch cards, as described, there is a much simpler alternative. To get the sum of the variables for any variate or the sum of the squares, we need only sequence sort the variate and cumulative-multiply on a desk calculator the number of cards in each sort by the cell number or square of the cell number. Both operations may be made together. To get the sum of the cross-products of a pair of variates we can sequence-sort for one of the variates and then sequence-sort each of the above sorts for the cell numbers of the second variate. Again we would count the number of cards in each sort and cumulative-multiply these counts by the product of the two cell numbers which identify the sort.

The preceding calculational scheme can be used to determine the elements of the covariance matrices which arise in the least squares analysis of non-orthogonal variance designs. Since in these cases each observable variate will have a value of either zero or one, a sum of the observations for a given variate or a sum of the squared observations will simply be the number of cards which have been notched on the given variate; similarly a sum of the cross-products of a given pair of variates will be the number of cards which have been notched on the given pair of variates.

If the covariance matrices are not large, they can be inverted with the aid of a desk calculator and an efficient computational scheme for inverting matrices. [3] If the matrix is too large for the above method of inversion, it would be necessary to have the matrix inverted on a high-speed programmed computer. In either case the marginal punch card system can save a great deal of time in the preparation of these matrices.

CONCLUSION

While marginal punch card systems provide fast and economic means for sorting relatively large amounts of data, they are not as suitable as the IBM or Remington Rand systems for performing numerical calculations on these data. However, in general only classificatory data are encoded and when these are calculated, all that is usually necessary is to sort and then follow by a simple counting operation.

Usually numerical observations, such as the accepted and rejected (xx'd) yardages in Fig. 2, are written on the face of the cards and not encoded. The reason is that, if these data were sequence sorted, most sorts would include one or two cards so that a grouped method of calculation would not be likely to pay for the encoding and sorting efforts. However, such data might profitably be encoded if it were possible to group the data and to encode only cell numbers. For example, the previous yardages might be mentally rounded to the nearest ten and encoded as multiples of ten.

This discussion of the uses of marginal punch cards has had to be brief. However, it is hoped that this discussion will serve to introduce the references on these methods and to encourage readers to try them out.

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1. *Punched Cards (Their Application to Science and Industry)*, edited by Robert S. Casey and James W. Perry, Reinhold Publishing Co., 1951.
2. *Application of Punch Cards in the Analysis of Textile Defects*, Oliver P. Beckwith, Baltimore Proceedings, A.S.Q.C., 1954.
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Some Applications of Hotelling's T

CHARLES R. HICKS

Purdue University, Lafayette, Ind.

A problem often encountered in industry is to test whether or not two sample means differ by more than one would normally expect by chance. Is the average diameter of parts produced by machine A significantly different from the average diameter of parts produced by machine B? Or, can we claim that two samples of light bulbs came from different lots when we observe a difference in mean life of 120 hours? The usual technique used to help answer such questions is the t test for testing whether or not two samples might have been chosen from the same normal population in regard to some measured quality characteristic. The t table will enable us to make some probability statement about our getting the observed difference in means when we make the hypothesis that both samples came from the same normal population. If the observed t value exceeds the five-percent t value, we usually reject our hypothesis and conclude that the observed difference in means is a real difference and did not just arise from random sampling errors.

Now suppose we measure not one, but two quality characteristics or variables on each sample. What can be said about the difference in the means of the two samples on each of these variables? In 1931, Hotelling[1] formulated a generalized t test which can be applied to this problem of testing the difference in means between two groups on several variables acting simultaneously. This statistic is called T and can be evaluated from the sums, sums of squares and sums of cross products of the data.

Consider a problem in the manufacture of low carbon M iron being made on two successive shifts as to whether or not there is a significant difference in the product from the two shifts with respect to percent manganese (% Mn) and percent chromium (% Cr). Taking a sample of 50 pieces from one shift and 49 pieces from the other, the summary data in Table I were found (after coding). In Table I, the X's refer to the 1st shift data and the Y's to the 2nd shift data. The subscript indicates which variable is measured, a one being used to identify the % Mn and a two the % Cr. If we define $x = X - \overline{X}$, then in order to calculate the usual t statistic we compute the entries in Table II from those in Table I.

TABLE I

	1st Shift ($N_1 = 50$)	2nd Shift ($N_2 = 49$)
% Mn ($=X_1$)	$\Sigma X_1 = 7.437$ $\Sigma X_1^2 = 1.214.013$	$\Sigma Y_1 = 6.866$ $\Sigma Y_1^2 = 1.070.024$
% Cr ($=X_2$)	$\Sigma X_2 = 1.498$ $\Sigma X_2^2 = 48.974$	$\Sigma Y_2 = 1.613$ $\Sigma Y_2^2 = 56.607$

TABLE II

	1st Shift	2nd Shift
% Mn ($=X_1$)	$\overline{X}_1 = 148.74$ $\Sigma x_1^2 = 107.834$	$\overline{Y}_1 = 140.12$ $\Sigma y_1^2 = 107.960$
% Cr ($=X_2$)	$\overline{X}_2 = 29.96$ $\Sigma x_2^2 = 4.094$	$\overline{Y}_2 = 32.92$ $\Sigma y_2^2 = 3.507$

(Here $\Sigma x^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{N}$)

To test the difference in the means for Mn we use

$$t_1 = \frac{\overline{X}_1 - \overline{Y}_1}{\sqrt{\left(\frac{\Sigma x_1^2 + \Sigma y_1^2}{N_1 + N_2 - 2}\right)\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}} \qquad (1)$$

with $n = N_1 + N_2 - 2$ degrees of freedom; a similar formula is used for the second variable.

It will be recalled that t represents the ratio of the observed difference in sample means to the standard error of the difference in sample means where this latter is estimated from the sample data. The standard error thus acts as a yardstick against which to compare the observed difference in means.

Substituting the data from Table II above

$$t_1 = \frac{148.74 - 140.12}{\sqrt{\left(\frac{107{,}834 + 107{,}960}{50 + 49 - 2}\right)\left(\frac{1}{50} + \frac{1}{49}\right)}} = 0.92$$

with $n = N_1 + N_2 - 2 = 97$ d. f.

$$t_2 = \frac{29.96 - 32.92}{\sqrt{\left(\frac{4{,}094 + 3{,}507}{50 + 49 - 2}\right)\left(\frac{1}{50} + \frac{1}{49}\right)}} = -1.66$$

also with 97 d. f.

For 97 d. f., the five-percent value is 1.99, so we would be led to conclude that there is no significant difference in the average of either the % Mn or % Cr in low carbon iron from the two shifts. We do not reject the null hypothesis; that is, the evidence in these data for a difference between shifts is not conclusive.

To apply Hotelling's T to these data we define the following:

$$d_1 = \frac{\overline{X}_1 - \overline{Y}_1}{\sqrt{\frac{1}{N_1} + \frac{1}{N_2}}}$$

$$\text{and} \quad d_2 = \frac{\overline{X}_2 - \overline{Y}_2}{\sqrt{\frac{1}{N_1} + \frac{1}{N_2}}} \tag{2}$$

$$a_{11} = \frac{1}{n}\left[\Sigma x_1^2 + \Sigma y_1^2\right]$$

$$a_{12} = \frac{1}{n}\left[\Sigma x_1 x_2 + \Sigma y_1 y_2\right] \tag{3}$$

$$a_{22} = \frac{1}{n}\left[\Sigma x_2^2 + \Sigma y_2^2\right]$$

where n is the number of degrees of freedom equal to $N_1 + N_2 - 2$ and the summations are over all the x's in a given sample.

From these we formulate

$$T^2 = \frac{d_1^2 a_{22} - 2d_1 d_2 a_{12} + d_2^2 a_{11}}{a_{11}a_{22} - a_{12}^2} \tag{4}$$

The statistic T^2 is thus a function of the differences in means and unbiased estimates of the variances and covariances based on the within-sample variations. T^2 is actually then a variance ratio and its significance can be tested with an F test. T^2 reduces to t^2 when only one variable is involved.

In the problem above, we have all of the ingredients for this formula except $\Sigma x_1 x_2$ and $\Sigma y_1 y_2$. These are easily computed from the original data

$$\Sigma X_1 X_2 = 235{,}215$$
$$\Sigma Y_1 Y_2 = 238{,}844$$

from which

$$\Sigma x_1 x_2 = \Sigma X_1 X_2 - \frac{(\Sigma X_1)(\Sigma X_2)}{N_1}$$

$$= 235{,}215 - \frac{(7{,}437)(1{,}498)}{50}$$

$$= 12{,}402$$

and

$$\Sigma y_1 y_2 = \Sigma Y_1 Y_2 - \frac{(\Sigma Y_1)(\Sigma Y_2)}{N_2}$$

$$= 238{,}844 - \frac{(6{,}866)(1{,}613)}{49}$$

$$= 12{,}830$$

Substituting into formulas (2) through (4)

$$d_1 = \frac{148.74 - 140.12}{\sqrt{\frac{1}{50} + \frac{1}{49}}} = 42.8819$$

$$d_2 = \frac{29.96 - 32.92}{\sqrt{\frac{1}{50} + \frac{1}{49}}} = -14.7251$$

$$a_{11} = \frac{1}{97}\left[107{,}834 + 107{,}960\right] = 2{,}224.6812$$

$$a_{12} = \frac{1}{97}\left[12{,}402 + 12{,}830\right] = 260.1332$$

$$a_{22} = \frac{1}{97}\left[4{,}094 + 3{,}507\right] = 78.3604$$

and

$$T^2 = \frac{(42.8819)^2(78.3604) - 2(42.8819)(-14.7251)(260.1322) + (-14.7251)^2(2224.6812)}{(2{,}224.6812)(78.3604) - (260.1332)^2}$$

$$= 8.95 \text{ with } 97 \text{ d. f.}$$

In order to test the significance of this result, we use the fact that

$$F = \left(\frac{n - p + 1}{np}\right) T^2 \tag{5}$$

where p is the number of variables involved (p=2, here) and enter an F table with $n_1 = p$; $n_2 = n - p + 1$ d. f.

For this problem

$$F = \frac{(97 - 2 + 1)}{(97)\ (2)}(8.95) = 4.44$$

with $n_1 = 2$;

$n_2 = 96$ d.f.

From an F table, the five-percent F for these degrees of freedom is $F_{.05} = 3.09$. This means then that there is a significant difference between the shifts with regard to the two variables % Mn and % Cr.

The apparent contradiction here in using two separate t tests and the single T test is due to the fact that T accounts for the possible correlation between % Mn and % Cr within each of the samples and hence gives a proper test of the difference in the means of both variables acting simultaneously as they actually do in contributing to the quality of the product.

To help clarify these results of t and T tests, T^2 can be expressed in terms of the t's and r—the average within-groups correlation coefficient. For the two-variable case above

$$T^2 = \frac{t_1^2 - 2rt_1t_2 + t_2^2}{1 - r^2} \tag{6}$$

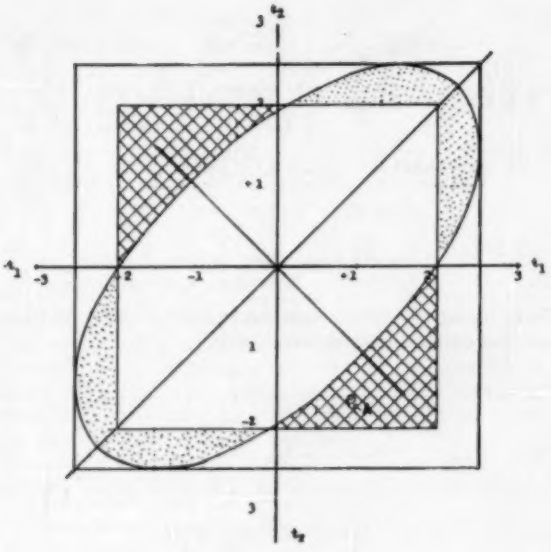

Figure 1.

For a fixed correlation (which is 0.623 on the above data) and the five-percent T^2 value (here equal to 6.24) this expression, formula (6), is a function of t_1 and t_2.[2] In fact it can be shown to be an ellipse in a t_1,t_2 coordinate system with its major axis inclined 45°, lengths of axes $2T_0\sqrt{1+r}$, $2T_0\sqrt{1-r}$, and inscribed in a square with side of length $2T_0$. Here T_0 is the desired significance point of T; i.e. $T_0 = \sqrt{T_0^2} = \sqrt{6.24} = 2.5$ in our problem.

For the data above, the diagram is roughly that shown in Fig. 1.

The area outside the ellipse gives values of T that are significant at the five-percent level and the area outside the inner square indicates where *both* t values are significant at this level. The dotted areas in Fig. 1 indicates regions where both t values are significant but T is not significant; the cross-hatched areas show where T is significant and neither t value is significant. If our t values of 0.92 and −1.66 are plotted as a point A (t_1,t_2) or (.92, −1.66) on Fig. 1 it is seen that although the point is well within the inner square it is *outside* the ellipse and hence only the T is significant. The size of these areas where the t's are significant and T not (and vice-versa) depend on the value of the average within-groups correlation.

From this example, one sees the advantage of using Hotelling's T instead of individual t tests when testing the difference in means of two groups on two variables acting simultaneously.

This technique can be extended to test the difference in means of two groups on several variables by extending the above formulas. The actual solving of problems of 5, 6, or more variables becomes quite laborious unless special forms are devised for solving the higher order determinants involved in the solution. An example will illustrate the use of this T technique although the details of the solution will be omitted.

Corn steep is purchased from two vendors, A and B, and samples from each vendor's lots are tested for six different variables: X_1 = pH, X_2 = Total % Solids, X_3 = % Fat-Ether Extractable, X_4 = Iodine Number, X_5 = Total % Nitrogen, X_6 = Lactic Acid. The hypothesis is that there are no differences in means of the six variables. We wish then to test whether or not there is any significant difference at the five-percent level between the products of the two vendors considering all six variables acting together.

Six sample batches from each vendor were tested alternately with the results shown in Table III.

TABLE III—Chemical Properties of Corn Steep from Two Vendors: A and B

Steep No.	pH	Total % Solids	% Fat-Ether Extractible	Iodine No.	Total % Nitrogen	Lactic Acid
Group I—Vendor A's Corn Steep						
	X_1'	X_2'	X_3'	X_4'	X_5'	X_6'
1	3.8	52.0	0.07	59.9	3.99	24.8
3	3.9	53.9	0.62	100.7	4.14	21.4
5	4.2	51.8	0.25	81.7	4.07	22.2
7	4.0	48.0	0.29	79.8	3.73	25.6
9	4.0	48.7	0.52	88.3	4.16	24.0
11	3.9	50.6	0.12	63.8	3.87	24.9
Group II—Vendor B's Corn Steep						
	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
2	3.9	51.2	0.39	79.1	3.86	31.0
4	4.0	53.5	0.34	63.0	3.71	30.7
6	4.0	49.8	0.87	89.1	3.86	26.8
8	4.4	49.8	0.31	79.8	4.37	26.8
10	4.0	47.1	1.27	73.2	4.19	28.1
12	4.0	50.9	0.04	40.8	4.11	30.3

To calculate T^2, extend formula (2) above to include six variables, giving $d_1, d_2, \ldots d_6$ and extend (3) to include six variables taken two at a time giving 36 a_{ij} values. Then T^2 can be written as the ratio of two determinants:

$$T^2 = - \frac{\begin{vmatrix} 0 & d_1 & d_2 & \cdots & d_6 \\ d_1 & a_{11} & a_{12} & \cdots & a_{16} \\ d_2 & a_{21} & a_{22} & \cdots & a_{26} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ d_6 & a_{61} & a_{62} & \cdots & a_{66} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{16} \\ a_{21} & a_{22} & \cdots & a_{26} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ a_{61} & a_{62} & \cdots & a_{66} \end{vmatrix}} \qquad (7)$$

This formula can be applied to as many variables as appear relevant in a given problem, and the significance of T^2 tested by using formula (5) and the F table. In this problem p = 6 since six variables are involved.

After the calculations are made following the above formulas, $T^2 = 186$; a value which is highly significant even at less than the one-percent level of significance and it is safe to conclude that the corn steep from the two vendors is of considerably different quality as measured by all six of these variables acting together.

MATHEMATICAL APPENDIX

We shall now show just how T is related to the individual t's and the intercorrelations between the p

variables. Hotelling formulates T^2 in one form as

$$T^2 = -\frac{\begin{vmatrix} 0 & d_1 & d_2 & \cdots & d_p \\ d_1 & a_{11} & a_{12} & \cdots & d_{1p} \\ d_2 & a_{21} & a_{22} & \cdots & d_{2p} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ d_p & a_{p1} & a_{p2} & \cdots & a_{pp} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ a_{p1} & a_{p2} & \cdots & a_{pp} \end{vmatrix}} \qquad (7')$$

(A generalization of formula 7.) From formula (1) in general:

$$t_i = \frac{\overline{X}_i - \overline{Y}_i}{\sqrt{\left(\frac{\Sigma x_i^2 + \Sigma y_i^2}{N_1 + N_2 - 2}\right)\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}} \qquad (8)$$

and from (2) and (3):

$$d_i = \frac{\overline{X}_i - \overline{Y}_i}{\sqrt{\frac{1}{N_1} + \frac{1}{N_2}}} \qquad (9)$$

$$a_{ij} = \frac{1}{n}\left[\Sigma x_i x_j + \Sigma y_i y_j\right] \qquad (10)$$

where $n = N_1 + N_2 - 2$

Substituting (9) and (10) in (8):

$$t_i = \frac{d_i}{\sqrt{a_{ii}}} \text{ or } d_i = t_i\sqrt{a_{ii}} \qquad (11)$$

The average within groups correlation coefficient is

$$r_{xy} = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}}$$

or

$$r_{ij} = \frac{\Sigma(x_i x_j + y_i y_j)}{\sqrt{\Sigma(x_i^2 + y_i^2)\ .\ \Sigma(x_j^2 + y_j^2)}} \qquad (12)$$

Substituting (10) in (12)

$$r_{ij} = \frac{a_{ij}}{\sqrt{a_{ii}\ .\ a_{jj}}} \text{ or } a_{ij} = r_{ij}\ \sqrt{a_{ii}a_{jj}} \qquad (13)$$

Now substituting (11) and (13) into (7')

$$T^2 = -\frac{\begin{vmatrix} 0 & t_1\sqrt{a_{11}} & t_2\sqrt{a_{22}} & \cdots & t_p\sqrt{a_{pp}} \\ t_1\sqrt{a_{11}} & r_{11}\sqrt{a_{11}a_{11}} & r_{12}\sqrt{a_{11}a_{22}} & \cdots & r_{1p}\sqrt{a_{11}a_{pp}} \\ t_2\sqrt{a_{22}} & r_{21}\sqrt{a_{22}a_{11}} & r_{22}\sqrt{a_{22}a_{22}} & \cdots & r_{2p}\sqrt{a_{22}a_{pp}} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ t_p\sqrt{a_{pp}} & r_{p1}\sqrt{a_{pp}a_{11}} & r_{p2}\sqrt{a_{pp}a_{22}} & \cdots & r_{pp}\sqrt{a_{pp}a_{pp}} \end{vmatrix}}{\begin{vmatrix} r_{11}\sqrt{a_{11}a_{11}} & r_{12}\sqrt{a_{11}a_{22}} & \cdots & r_{1p}\sqrt{a_{11}a_{pp}} \\ r_{21}\sqrt{a_{22}a_{11}} & r_{22}\sqrt{a_{22}a_{22}} & \cdots & r_{2p}\sqrt{a_{22}a_{pp}} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ r_{p1}\sqrt{a_{pp}a_{11}} & r_{p2}\sqrt{a_{pp}a_{22}} & \cdots & r_{pp}\sqrt{a_{pp}a_{pp}} \end{vmatrix}}$$

Factoring out terms common to given columns in each determinant and substituting $r_{11} = r_{22} = \ldots = 1$

$$T^2 = -\frac{-\sqrt{a_{11}}\sqrt{a_{22}}\cdots\sqrt{a_{pp}}\begin{vmatrix} 0 & t_1 & t_2 & \cdots & t_p \\ t_1\sqrt{a_{11}} & \sqrt{a_{11}} & r_{12}\sqrt{a_{11}} & \cdots & r_{1p}\sqrt{a_{11}} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ t_p\sqrt{a_{pp}} & r_{p1}\sqrt{a_{pp}} & r_{p2}\sqrt{a_{pp}} & \cdots & \sqrt{a_{pp}} \end{vmatrix}}{\sqrt{a_{11}}\sqrt{a_{22}}\cdots\sqrt{a_{pp}}\begin{vmatrix} \sqrt{a_{11}} & r_{12}\sqrt{a_{11}} & \cdots & r_{1p}\sqrt{a_{11}} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ r_{p1}\sqrt{a_{pp}} & r_{p2}\sqrt{a_{pp}} & \cdots & \sqrt{a_{pp}} \end{vmatrix}}$$

Now, factor out terms common to rows of both numerator and denominator determinants

$$T^2 = -\frac{-a_{11}a_{22}\cdots a_{pp}\begin{vmatrix} 0 & t_1 & t_2 & \cdots t_p \\ t_1 & 1 & r_{12} & \cdots r_{1p} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ t_p & r_{p1} & r_{p2} & \cdots 1 \end{vmatrix}}{a_{11}a_{22}\cdots a_{pp}\begin{vmatrix} 1 & r_{12}\cdots r_{1p} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ r_{p1} & r_{p2}\cdots 1 \end{vmatrix}}$$

$$= -\frac{\begin{vmatrix} 0 & t_1 & t_2 & \cdots t_p \\ t_1 & 1 & r_{12} & \cdots r_{1p} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ t_p & r_{p1} & r_{p2} & \cdots 1 \end{vmatrix}}{\begin{vmatrix} 1 & r_{12}\cdots r_{1p} \\ r_{21} & 1 & \cdots r_{2p} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ r_{p1} & r_{p2}\cdots 1 \end{vmatrix}} \qquad (14)$$

which gives T^2 in terms of the t's and the intercorrelations. For the special case where $p = 2$ variables, this becomes

$$T^2 = -\frac{\begin{vmatrix} 0 & t_1 & t_2 \\ t_1 & 1 & r_{12} \\ t_2 & r_{21} & 1 \end{vmatrix}}{\begin{vmatrix} 1 & r_{12} \\ r_{21} & 1 \end{vmatrix}}$$

or expanding

$$T^2 = \frac{t_1^2 - 2r_{12}t_1t_2 + t_2^2}{1 - r_{12}^2}$$

which is formula (6) above.

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1. Hotelling, Harold, "The Generalization of Student's Ratio," *Annals of Mathematical Statistics*, II (1931), pp. 360-378.
2. Hald, A., *Statistical Theory with Engineering Applications*, John Wiley and Sons, 1952, pp. 607-621.
3. Hicks, Charles R., *A Study of Hotelling's T Statistic and its Application to Problems in Psychology, Education and Industrial Research.* Unpublished Ph.D. Thesis, Syracuse University, August 1953.

Problems Department

PAUL C. CLIFFORD and THOMAS A. BUDNE, *Co-editors*

Problems for solution, and correspondence concerning material, published in this department, should be addressed to:
Thomas A. Budne
Philip Morris & Co., Ltd.
100 Park Ave.
New York 17, N. Y.

THE MORE THE MERRIER!

When we asked for diversified problems—practical, theoretical, and philosophical—and lots of them, we weren't quite prepared to expect the degree of cooperation which our readers have offered. Our only problem now, forgetting the one about sorting out answers, is getting your problems on paper fast enough to meet these new-fangled monthly deadlines.

However don't let this stop you. If you have any problems, we're entitled to have a couple of our own.

NEW PROBLEMS

Problem 15. Submitted by E. Terry Dalton, Mount Vernon, Washington.

In the production of frozen green peas, the peas are separated by density differences in a machine containing a salt brine. (Tenderness is inversely proportional to density.) The tenderest peas float and the average tenderness (density) of the "floaters" is controlled by adjusting the density of the salt brine. The proportion of peas that sink (or float) is also directly related to the density of the salt brine, and to the distribution of pea densities presented to the machine.

At the present time, we measure the density of the "floaters" and of the "sinkers" and control the "floater" density to a desired level, letting the "sinkers" density fall where it may. The "sinker" fraction amounts to about 10% of the peas put through the machine and tens of millions of dollars worth of product are thus handled in the industry annually.

The measure that we presently use to obtain pea density data is to test a sample of peas ($n = 50$) in a beaker of salt brine which has been carefully adjusted to standard density, determining the percentage of peas that sink in the test brine for the sample of 50. This will be recognized as an attribute method of testing and, although other methods are available, the speed and simplicity of the test argue the desirability of its use.

Using a sample of salt brine from the machine, it is possible to check incoming peas that will be presented to the machine to determine on a sample basis the expected fractions of "sinkers" and "floaters" at the discharge end of the device. It is also possible to use a sample of salt brine from the machine in which to test samples of the "floater" fraction discharged from the machine to determine on a sample basis what percentage of the peas the machine classed as "floaters" which should not have been. The "sinker" fraction of peas from the discharge of the machine can likewise be tested in the machine separation brine to determine the percentage of peas erroneously classed by the machine as "sinkers."

The tests just mentioned are performed by placing a sample of the particular peas in question in a beaker of the machine salt brine and the problem arises in the fact that certain peas that would float in this still brine of a given density, actually do not under machine operation. Brine in operation is not still, flowing through the machine to carry the peas with it, and producing adverse currents that carry tender peas into the "sinker" category (lower priced) while causing less tender peas to be included as "floaters" (lowering their average tenderness.)

How do we handle the data we are obtaining to obtain a measure of separation efficiency that can be used for process control as well as indicating machine capability?

Problem 16. Suggested by E. G. Field, Atlanta, Georgia.

How is the control chart constant c_2 derived and can you illustrate for some particular value of n?

Practical Aids

ELLIS R. OTT, *Editor*

Send your "Practical Aid" contributions to:
Dr. Ellis R. Ott
Rutgers University
77 Hamilton Street
New Brunswick, N. J.

Trouble-Shooting

ELLIS R. OTT

Rutgers University, New Brunswick, N. J.

INTRODUCTION

Excessive rejects were occurring in the assembly department (mounting) on a certain type of electronic tube. Several hundred of these (mounts) were being produced each day by teams of girls using spot-welding machines. Particular trouble was being experienced by one of these teams. The mount-assemblies were inspected after a succession of welds by different welders and it was difficult to identify the source of welding trouble. (In regular production, each girl was assigned to one specific welder. No attempt was made in this first

study to separate the effect of the operators from that of the machines.)

The department head believed that the trouble was all caused by substandard parts which were being delivered to the department. In fact, there was some evidence supporting this view—the foreman of the preceding operation agreed that the parts he was sending on to welding were inferior. Even so, any operation with more than three or four operator-machine combinations is worth at least a special study when it is in trouble.

This note discusses a straightforward approach to the type of problem described above, which is characterized by a succession of operations (a) which produce a product which can only be classified as "satisfactory" or "unsatisfactory," and (b) in which there are several different operators, or machines, or heads on a machine, or jigs and fixtures all doing the same operation. The procedure is to select small samples of the product for a *special study* and examine each one carefully, and then record this sample data for careful analysis. The procedure is described below. Experience has shown that these small samples, examined carefully, usually indicate more useful information for corrective action on the process than information obtained later from a 100 per cent inspection of the product.

COLLECTING DATA

A roving inspector was assigned (for three days) to pick five mounts from each operator-machine combination at approximately hourly intervals. Each weld was inspected immediately and a record of each type of weld-defect was recorded on a special record-form by operator-machine. Over the three-day period of the special study, records were obtained on 11 different operator-machine combinations; these combinations have been labelled A, B, C, ... , K in Fig. 1. The average percent of weld-rejects (all kinds combined) for the entire group for the three-day study was essentially eight percent. However, there was variation about this eight percent average; the highest (D) was 17 percent and the lowest was one percent. Three of the combinations (F, H, K) had relatively few rejects.

INTERPRETATION

There are several different known factors which might introduce trouble into this spot-welding operation:

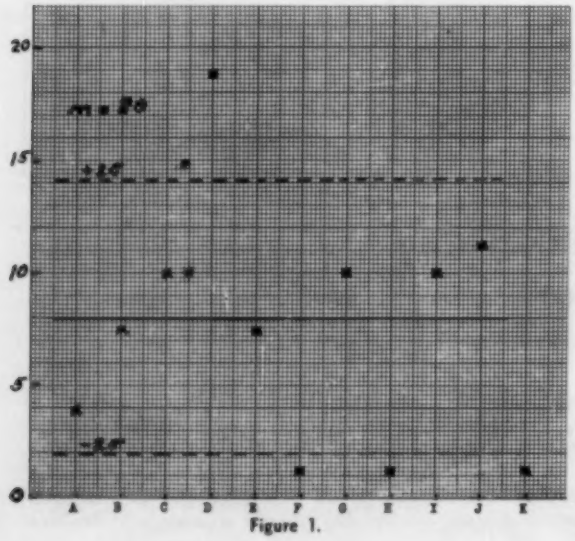

Figure 1.

(1) The parts being welded together

(2) The type of welding machine being used

(3) The maintenance of the machine

(4) The operators.

The first question is whether the variation observed in this set of data represents chance variation, or whether there is evidence of essential differences between the operator-machine combinations. During the three-day period, a total of 80 pieces from each combination, out of the many hundreds which had been made, had been inspected. The 3-σ control limits for samples of size n=80 and an average fraction defective p=0.08 are shown in Fig. 1. Combination D exceeded the upper 3-σ limit, and combinations F, H, and K were within 3-σ limits but below 2-σ limits.

In a trouble-shooting experiment of this kind we frequently choose 2-σ limits as a (tentative) basis for judging a departure (from average) to be sufficiently important to study. Consequently, we discussed the four combinations with the forelady who was cooperating in the study. She assured us that the operator in Combination B was both "slow and careless." The operator in F was a very fast operator and also a careful one; the operator in H was slow but careful; and the operator in K was one about whom she knew little because she was a "utility girl" who was moved about the plant as needed.

We can make the following conclusions: There are several factors contributing to the defects which were being produced. These include the possibility of poor parts, of particular machine-operator combinations, and extraneous things such as the day of the week and the weather. However, these small samples, when pooled, indicate that one machine-operator combination (B) is very bad; and although the parts are admittedly bad, three operators have found important ways of reducing their effect. This latter conclusion now presents a very important situation. The forelady believes that Operators F and H are effecting their improved welding by repeating the weld as many as seven or eight times instead of once as specified. If this is a reasonable way to perform the operation, then other operators should be instructed to make similar welds. It may be, however, that the repeat-welding, which gives improvement at this particular location, may be introducing excessive failures at later stages in the assembly of the radio tubes. Neither this question—nor whether the operators, the welding-machines, or the parts are most important factors—can be answered by the available data. There is now reason, however, to plan a scientifically designed experiment which can help us reach an economically satisfactory answer.

CONCLUSIONS

Pooling the (attribute) information from small samples (of five each) over a three-day period indicates important welding differences in operator-machine combinations which was independent of the quality of parts being delivered to the department. These differences suggest that: (a) Combinations F, H, and K should be watched for clues to their successful techniques, in the hope that they may be taught to the other operators, and (b) combination D should be watched to check the forelady's impression of her. This visual analysis may supply the desired information. If not, then these differences suggest the desirability of designing a scientific study of the problem, and indicate important factors to include in the experiment. A conference of pro-

duction and engineering supervision can decide which of the following factors, and the number of "levels" of each factor to include:

(a) Spot-welders (certainly include the one in combination D and one from F, H, or K)

(b) Operators

(c) Parts

(d) Spot-welder maintenance.

Examples of simple experimental designs which have proved effective in production trouble-shooting are described in the following articles:

"A Production Experiment with Mechanical Assemblies," *Industrial Quality Control*, September, 1952, p. 25; E. R. Ott.

"A Production Experiment Using Attribute Data," *Industrial Quality Control*, March, 1952, p. 41; Ennerson, Fleischmann, Rosenberg.

"Forty Little Bottles and How They Fell," *Drug and Cosmetic Industry*, December, 1954; Francis A. Mina & Mason E. Wescott.

Bibliography

JOSEPH MOVSHIN, *Editor*

Assisted by the Editorial Committee, St. Louis Section, ASQC

On matters concerning the Bibliography Department, send correspondence, suggestions, and contributions to:

Joseph Movshin
9220 Old Bonhomme Road
St. Louis 24, Mo.

APPLICATION—METAL WORKING

8TH ANNUAL QUALITY CONTROL FEATURE ISSUE
(Tooling and Production, v. XXI, n. 2, May 1955)

Several articles in this issue describe the application of a quality control system in general terms of a particular product. The importance of Quality Control and the benefits derived generally are stressed. An editorial stressing the growing importance of machine capability measurements is presented in this issue.

EVALUATING THE RESULTS OF A QUALITY CONTROL SYSTEM IN THE PLANT, by L. Grant Hector (Sonotone Corp.) Page 6

QUALITY CONTROL OF QUALITY CARS page 73

CADILLAC'S COMMUNICATIONS IN QUALITY MANUFACTURING, by Stanley B. Sherwood (Cadillac Motor Car Div., General Motors Corp.) page 74.

THE IMPERIAL—A CHRYSLER CONCEPT OF QUALITY CONTROL, by Robert W. Lotz (Chrysler Div., Chrysler Corp.) page 76

QUALITY—CONTROLLING LINCOLN'S NEW TURBO-DRIVE, by Roscoe M. Smith (Ford Motor Co.) page 78

PACKARD'S 'TORSION LEVEL RIDE'—A STUDY IN QUALITY CONTROL, by B. P. Seiple (Studebaker-Packard Corp.) page 80

WHY A QUALITY CONTROL SYSTEM WITH CONTROL CHARTS? by Frank Caplan, Jr. (General Electric Co.) page 88

SHORT CUTS IN PROCESSING QUALITY CONTROL DATA, by Michael P. Agresta (Pesco Products Div., Borg-Warner Corp.) page 92

Charts for control limits for different tolerances are given.

9TH ANNUAL CONVENTION AMERICAN SOCIETY FOR QUALITY CONTROL page 96.

STEEL BUYING BY STATISTICS, by Thomas F. Hruby
(Steel—v. 136, n. 16, Apr. 18, 1955, p. 104)

This article recommends the plotting of test results on all incoming steel to prevent trouble in production and to assure receiving the proper material. Frequency distributions on many basic characteristics in the form of histograms are collected.

APPLICATION—MISCELLANEOUS

QUALITY CONTROL, by W. Wotus
(Materials & Methods—v. 41, n. 2, Feb. 1955, p. 96)

This article describes the Quality Control program at the Aero Supply Mfg. Co. The system is illustrated pictorially from the original delivery of coil stock material to the shipment of the finished bolts.

PUBLIC UTILITY BILLING—AN IDEAL SPOT FOR S.Q.C., by Walter W. Merrill
(The Edison Electric Institute Bulletin—v. 22, n. 5, May 1954)

This article points out the advantages of using sampling methods in checking public utility billings. The author also lists the factors that must be considered and sold to management prior to using a sampling program. Control limits on an np chart are used for detecting assignable causes.

MANAGEMENT & PERSONNEL, GENERAL

ANSWERS TO EVERYDAY QUESTIONS ABOUT QUALITY CONTROL
(The Foreman's Digest—v. 5, n. 4, Apr. 1955, p. 25)

The article describes the basic concepts of Quality Control in very general terms. It is primarily a non-technical description of the techniques as well as a presentation of the approaches that would be most interesting to operating personnel.

MISCELLANEOUS

QUALITY CONTROL SETS FACTORY TRAFFIC LIGHTS
(Electronics—v. 27, n. 10, Oct. 1954)

The use of red, orange, or green lights for the production line at Tung-Sol Electric Inc. is discussed in the Production Techniques section of this issue. The appropriate light is turned on by the Quality Control department on the basis of the inspection of current production. The method used to record the data is a special rolled strip chart.

STATISTICS

MULTIPLE RANGE AND MULTIPLE F TESTS, by David B. Duncan
(Biometrics—v. II, n. 1, Mar. 1955)

DETERMINING THE FRUIT

COUNT ON A TREE BY RANDOMIZATION BRANCH SAMPLING, by Raymond J. Jessen
Biometrics—v. II, n. 1, Mar. 1955)
THE MEASUREMENT OF SEASONAL MOVEMENTS IN PRICE AND QUANTITY INDEXES, by Bruce D. Mudgett
(Journal of the American Statistical Association—v. 50, n. 269, Mar. 1955)
SOME STATISTICAL PROBLEMS IN RELATING EXPERIMENTAL DATA TO PREDICTING PERFORMANCE OF A PRODUCTION PROCESS, by T. W. Anderson
(Journal of the American Statistical Association—v. 50, n. 269, Mar. 1955)

OPERATIONS RESEARCH

THE MONTE CARLO METHOD, by Daniel D. McCracken (General Electric Company, Cincinnati, Ohio)
(Scientific American—v. 192, n. 5, May 1955, p. 90)

The Monte Carlo method is used to predict the outcome of a series of events, each of which has its own probability. This article outlines the method in terms of neutrons, needles, roulette wheels and furniture factories.

OPERATIONS RESEARCH AND MANAGEMENT PROBLEMS, by Wroe Alderson (Visiting Professor, Massachusetts Institute of Technology)
(Advanced Management—v. XX, n. 4, Apr. 1955, p. 14)

This article describes the basic techniques of operations research as applied to various types of management problems. The problems are related to marketing, shopping, and competition problems.

LINEAR PROGRAMMING, by Robert O. Ferguson, Methods Engineering Council, Pittsburgh, Pa.
(American Machinist—v. 99, n. 8, Apr. 11, 1955, p. 121)

Highlights of Linear Programming under the following sections:

What is Linear Programming?

Defined as a method for picking a best choice when choices exist. Describes differences between L.P. and other scheduling methods. Outlines basic information needed.

How one company saved 20% with L.P.

This company, making 3500 different standard products and many special ones in lot sizes ranging from 1 to 1,000,000 pieces, found this scheduling method produced direct dollar savings and better deliveries.

The Index Method and how it is used.

Works through a simple example to show how it was used to compare ideal machine time with alternate machine time.

Another Company used Index Method—Gained 10% output in screw machine division.

Shows the programming procedure for assigning orders to machines to minimize costs.

L.P. balances production and distribution costs.

A case study of a company with two plants in different cities and how they used the MODI method to schedule production and distribution.

MODI is the simplest formal method.

Used when all information can be converted to common units.

Calculation methods are shown.

Other methods-distribution, simplex and ratio analysis.

Brief descriptions and a look at future trends.

PRODUCTION SCHEDULING: AN OPERATIONS RESEARCH CASE STUDY, by Dr. Russell L. Ackoff
(Advanced Management—v. XX, n. 3, Mar. 1955, p. 21)

This article discusses the problem of selection of lot size to secure lowest production cost. This approach to economic lot size discusses in some detail the development of a basic formula for this purpose, assuming total number of pieces needed per year and other fixed factors. This is illustrated with figures showing the function of the formula which are developed.

REPORTED—NOT REVIEWED

ON THE AVERAGING OF DATA, by S. S. Stevens
(Science—v. 121, Jan. 28, 1955, p. 113)

What's New?

ROY A. WYLIE, *Editor*

News releases describing new products and literature should be sent to:
George R. Foster, Mng. Ed.
Room 6123 Plankinton Bldg.
161 W. Wisconsin Ave.
Milwaukee 3, Wis.

A table model of the versatile Instron Universal Testing Instrument is announced by the **Instrom Engineering Corporation,** 440 Hancock St., Quincy, Mass. This compact instrument records electronically the stress-strain curve of a sample under tension or compression. The instrument is adaptable for quality control or research in plastics, rubber, textile, wire, paper, etc. Full scale ranges from two grams to 200 pounds are available through the choice of three load cells. Inertialess weighing and elimination of mechanical friction allow a degree of accuracy not usually obtainable in an instrument of this type. The high speed recorder is equipped with a choice of synchronous chart speeds to provide a range of extension magnification. The crosshead is driven synchronously through magnetic clutches and change gears to provide a choice of constant and reversible testing speeds over a range of 250 to one. Range can be from 0.2 to 50 inches per minute or in an alternate model, 0.02 to five inches per minute with a jaw travel of 30 inches. An unobstructed space of three inches is provided around the cross head assembly to enable the use of special conditioning chambers, without modification of the unit. Accessory equipment is available to meet individual testing requirements.

★ ★ ★

The Niagara Blower Company, 405 Lexington Ave., New York 17, N. Y., has announced a completely air conditioned test cabinet to provide full control of relative humidity and temperature for testing materials, products or specimens. Air for the cabinet is saturated at the required dew-point temperature and reheated

to the desired dry-bulb temperature. Close control of relative humidity is obtained within the range of 5% to 95%. The dry-bulb temperature at which this control is provided will depend on the collateral services afforded it. With only water and electric power the low limit depends on the available water temperature and the high limit is approximately 150 degrees F. When refrigeration is provided the dew-point temperature may be taken as low as minus 30 degrees F. with a corresponding lowering of the dry-bulb temperature at which low humidities can be produced. No moisture-sensitive instruments are required to achieve and hold the control, and recording instruments may be used. Dimensions in the cabinet are 30 x 28 x 24 inches. There is a window for viewing the test material.

★ ★ ★

A new Sine-Line involute gear checker designated as Model 1124-P is announced by **Michigan Tool Company**, 7171 E. McNichols Rd., Detroit 12, Mich. This unit will check involute contours and tooth spacing of gears ranging from 0 to 24 inches in diameter on shafts up to 60 inches long. This new checker utilizes the sine-bar principle to develop the tracing motion for indicating true involute. Only one master disc, combined with the sine-bar, is needed to obtain the proper ratio between the master disc and the base diameter of the gear being checked. Setups from gear-to-gear are simple to make and require only a few seconds. Two adjustments are required; the sine-bar is set to the correct corresponding angular position as given by standard formulii or tables, and the indicator contact finger is set to the new base diameter. Both spur and helical gears can be checked while mounted on solid or tubular shafts. A simple accessory permits checking internal gears in the same manner as external gears.

★ ★ ★

The Chicago Apparatus Co., 1735 N. Ashland Ave., Chicago, Ill., has adapted to the industrial and scientific laboratory the idea of "ready-mixes" common to the housewife in her kitchen. The new solutions, called "Stansols," bear the manufacturer's guarantee for accuracy within one part in 1,000, and are standardized five times before packing. Standards solutions are widely used in industrial and chemical laboratories as reference solutions for testing chemical formulas in research and control work. The solutions are standardized to a strength against which the comparison can be made. Preparations of standard solutions in the laboratory may take from 1 hour in standardizing against a secondary standard to 3 days in standardizing against a primary standard. "Stansols" permit preparation of accurate volumetric solutions in a few minutes and at much less cost. The solutions are put up in unbreakable polyethylene bottles of uniform size. A heat-sealed membrane under the screw cap eliminates spillage and atmospheric contamination. "Stansols" may be stored indefinitely without deterioration until opened and used. Chemicals which tend to be unstable in solution are supplied in crystalline form. "Stansols" are marketed in a wide variety of solutions and any type of solution for a particular need can be developed.

★ ★ ★

A new automatic tester for the assembly line testing of accounting machines is being manufactured by the **American Automatic Typewriter Co.**, 2323 N. Pulaski Rd., Chicago 39, Ill. This tester delays the "feeding" of the next problem until the machine completes the cycling, or solution, of the preceding problem. The tester stores sufficient problems on its activating record to give each machine a 40 minute test. A separate record of problems tests the machine on the less complicated adding and subtracting functions in which no automatic cycling adjustment is required. Actual hand-key operation is simulated without the possibility of human error. Rubber-tipped key-bars are connected to individual pneumatic bellows that open and close according to the coded perforations on the problem record roll. The pneumatic system used in the tester is similar to the Auto-typist mechanism made by the American Automatic Typewriter Company for use in business offices to type routine letters automatically.

★ ★ ★

Central Scientific Co., 1700 Irving Park Rd., Chicago, Ill., is marketing a new coulometric titrator, designed especially for the determination of mercaptan sulfur in petroleum stocks and other organic liquids. This device was developed by the Standard Oil Company of Ind., but manufacture and sale of the instrument will be handled under exclusive license agreement by Central Scientific Co. The titrator, using the coulometric system of operation, is capable of measuring unusually low

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Reading to	1 minute
Magnification	30X
Protractor Ocular	360°
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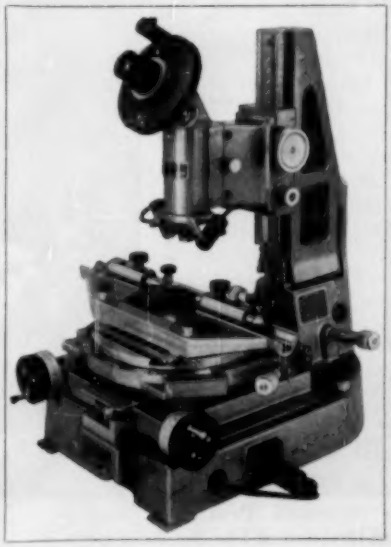


Write For Bulletin #147-50 and Index to Gaertner Literature

THE GAERTNER SCIENTIFIC CORPORATION

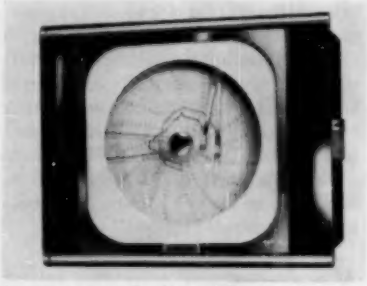
1248 Wrightwood Avenue
Chicago 14, Illinois

concentrations of mercaptan and provides the petroleum industry with a simpler and more accurate method for this important function in the control of refinery treating operations. A 1-ml sample is placed in a beaker containing two sets of electrodes: one is coulometric for generating silver ions; the other amperometric for detecting the end point. When the start button is pressed, the silver ions begin reacting with the mercaptans in the sample. At the end point, excess silver ions cause an increase in current flow between amperometric electrodes and stops the titration. The time of titration, which is easily read on a dial, gives a measure of the mercaptan content. On samples containing over 100 micrograms of mercaptan sulfur, the instrument is capable of obtaining a better than 2% agreement with potentiometer results. No standard solutions are required and titrations are very rapid. Fifteen to twenty samples of 1-ml each can be titrated in the cell before replacement of the electrolyte is necessary. The titrator is completely self-contained and operates from 115 volt, 60 cycle power supply.

★ ★ ★

The Standard Production Instrument Division of The Sheffield Corp., Dayton 1, Ohio, has developed a Plunjet gaging assembly for checking eight critical pump gear dimensions in one operation. The assembly consists of a Plunget supporting ring with rest pad, eight Plunjet units and two locating pins. The unit then checks the end play of the drive gear within limits of 0.0008 to 0.0018 inches, the end play of the driven gear within limits of 0.0008 to 0.0018 inches, and the average height of the crescent within limits of 0.0000 to plus 0.0003 inches. In operation the position of the floats in their respective Precisionaire columns indicate whether the dimensions and conditions are within tolerance or the amount they are out-of-tolerance.

★ ★ ★



A new round-chart Electronic Dynamaster Potentiometer recorder that can make two continuous records at the same time on a 12-inch round chart, has just been announced by **The Bristol Co.,** Waterbury 20, Conn. The dynamaster potentiometer and bridge instrumens can be used to measure and control any variable which can be translated into an electrical quantity, such as temperature, pH, consistency, speed, pressure, weight, and others. Comparison of two related measurements with respect to magnitude and time of any variations from the desired value is simpler and more accurate when a single chart is used than when two separate charts must be studied.

★ ★ ★

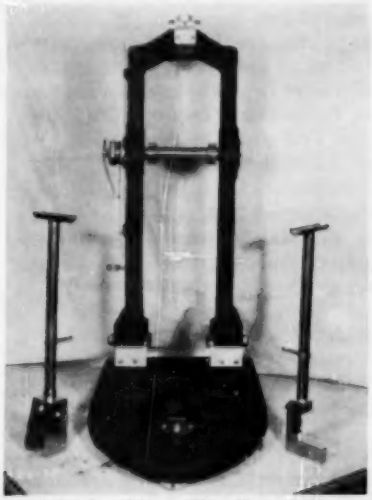
The Freeland Gauge Co., 9940 Freeland Ave., Detroit 27, Mich., has announced entry into the field of gage blocks and accessory sets. One of the first Freeland gage block sets to be introduced is the F-81. This consists of 81 special alloy steel blocks from 0.1001 to 4.000 inches. Sets are available in steel and chrome with accuracies of 0.000008, 0.000004, and 0.000002 inches, while protective end blocks are available in tungsten-carbide. Several other standard sets are available as well as an accessory set. Along with these, the Freeland company offers a gage block inspection service which guarantees the removal of imperfections from and the restoration of accuracy to gage blocks.

★ ★ ★

A new tester for measuring hardness and lubricity of materials, factors relating to measurement of resistance to scoring, gouging and abrasion of materials has just been announced by the **Taber Instrument Corp.,** Section 222, 111 Goundry St., North Tonawanda, N. Y. The new Dyhedron instrument will measure the hardness and lubricity of metals, glazed tile, glass, carbon, graphite, precast cement shapes, abrasive wheels and hones and like materials. The action of the diamond penetrator of this instrument differs from

the familiar stationary steel ball and conical diamond penetrators. The Dyhedron is a dynamic hardness tester utilizing a selected octahedral diamond shaped to work itself into the test material with a rotary oscillating motion. Its scope of testing ranges from soft plastic to tough steel, both solids and bonded granular materials. Depth of penetration is read from a precision dial indicator and the 360 degree oscillations of the diamond are read from the rotary counter.

★ ★ ★



Baldwin-Lima-Hamilton Corp., Philadelphia 42, Pa., has introduced a new impact test machine based on the Bell Telephone Laboratories design. Either Izod or Charpy tests can be made with 1 ft. lb. pendulum assemblies, as well as with other assemblies of 2, 4, 8, and 16 ft. lb. capacities. Pendulum assemblies, too, are quickly exchanged by means of a sliding collar arrangement. The machine meets latest specifications of A.S.T.M. and Federal Specifications for Organic Plastics.

★ ★ ★

A series of shock resistant light beam galvanometers of exceptional compactness and sensitivity are announced by **Howell Instrument Co.**, 1106 Norwood, Fort Worth, Tex.

Sensitivity ranges up to 0.105 microamperes per millimeter division. The effective length of the light beam is 80 millimeters. The unit will resist up to 25 G's shock or vibration. Overall size is 2.6 x 3.62 x 3.615 inches. Coil resistances are 20, 100, 500 and 1000 ohms. It is designed for flush or surface mounting and is of sealed construction to permit reliable operation in presence of dust or moisture. These galvanometers were originally developed to withstand the extremely severe conditions of shock and vibration encountered in field servicing and testing of jet aircraft. They are, however, well-suited for laboratory and industrial production testing services in the fields of colorimetry and densitometry as well as for measurement of electrical potentials, conductivity, light flux or temperature.

★ ★ ★

New Literature

The following new literature is available by writing direct to the company listed. Please mention *Industrial Quality Control* when you contact any of the companies mentioned in the "What's New?" department.

A 16-page brochure on the computational facilities and code library of Midwest Research Institute, 425 Volker Blvd., Kansas City, Mo., is now available. The brochure describes how the facilities can be used by firms throughout the country and the specialized equipment available to handle all types of problems.

A brochure which describes a new type of membrane filter for use in research, production and quality control is now offered by Millipore Filter Corp., Watertown 72, Mass. The booklet gives detailed information on use of the filter in microbiological studies; public health surveys; analysis of acid mist, metal fumes and smokes; quality control in food and beverage industries; research in drugs and pharmaceuticals and other areas of application.

Bausch & Lomb Optical Co., 635 St. Paul St., Rochester 2, N. Y., has published a new catalog which demonstrates the value of three-dimensional microscopes for industrial assembly lines, research laboratories, and quality control operations.

A booklet entitled "Work Simplification" by Ben S. Graham is now available from The Standard Register Co., Dayton 1, Ohio. This is a reprint of a series of articles from "Paperwork Simplification."

"Gamma Radiography" is the title of a booklet available from Technical Operations Inc., 6 Schouler Court, Arlington 74, Mass. This booklet describes techniques and equipment for gamma radiography in industrial inspection and quality control.

A new bulletin, "Temperature Control Systems," is now available from the Barber-Colman Co., Rockford, Ill. The bulletin contains an explanation of the various types of control systems, control terminology and rules to use as a guide for the selection of a proper method of control for process characteristics.

"Producing Leakproof Aluminum Castings" is the title of a booklet offered by George Sall Metals Company, 2300 E. Butler Street, Philadelphia 37, Penn. The article suggests operational methods by which foundrymen can produce castings of intricate design and high quality to meet rigid specifications.

Statistical Methods for Scientists in Industry Course

April marked the conclusion of a tutorial course consisting of 24 lectures on statistical methods held at the Management House, London, England. The course, attended by 16 technically qualified men in executive or research positions representing eight firms, enabled them to acquire a working knowledge of statistical methods, in particular the efficient planning of experiments and the analysis of data.

The lecturer, E. D. von Rest, planned the course around C. H. Goulden's main text-book, "Statistical Analysis."

As a result of the successful conclusion of the course, it was decided to supplement it with a short residential course to be held at Cambridge in the early summer on practical statistical calculations. Since the course is residential, its appeal to some who were unable to attend the London lectures has been offset by the provision for a modified course running parallel to the main course. This secondary course contains a shortened syllabus suitable for those beginning their study of the subject.

Further information on the summer course may be obtained by writing to: The Organiser; 18, Nightingale Road; Hampton, Middlesex, England.

Local Section News

". . . BUT NOT FORGOTTEN"

ALBUQUERQUE A panel discussion on "Problems in the Use of Quality Control" was the feature of the Apr. 6 meeting of the Aubuquerque Section held at the University of New Mexico. Results of the recent election were also announced.

BALTIMORE The Apr. 20 meeting held at Johns Hopkins University featured Dr. W. Edwards Deming's talk on "Random Pointers on Sampling." His keen ability to reach the crux of a problem coupled with his wit had many members taking a second look at their control plans the next morning. The members were thoughtfully provoked to consider how samples are taken; how many samples to take; how to interpret the results; and what margin of your answer is sampling error. Dr. Deming, a three time visitor to the Baltimore Section, insured his continued following by the stimulating and specific comments on sampling.

BUZZARDS BAY The monthly meeting of the Buzzards Bay Section was held in the Lecture Auditorium of Berkshire-Hathaway, Inc.

Mr. Leon Bass, Quality Control Manager, of the Jet Engine Dept., General Electric Co., Cincinnati, Ohio, and also Treasurer of the Society, was guest speaker. His topic, "Quality Control in the Electronics Industry," was very informative.

CLEVELAND The Cleveland Section was visited by William J. Youden of the National Bureau of Standards at its regular monthly meeting on Apr. 1. The title of his subject was "Statistics For Skeptics" which proved to be a very enlightening talk.

DAYTON The Dayton Section journeyed to Middletown, Ohio for its regularly scheduled meeting for April. An interesting plant tour of the Armco Steel Corp. was attended by one of the largest groups this season—a total of 80 consisting of 44 members and 36 guests. Dinner was served at the Manchester Hotel, courtesy of Armco Steel Corp. Following the dinner a representative of Armco discussed the use of quality control at Armco and gave a brief talk on the history and origin of Armco.

EVANSVILLE-OWENSBORO On Apr. 19 the Evansville-Owensboro Section met at the Sterling Brewery Rathskellar for a general session of good fellowship. Sampling was random and product quality excellent. Attendance was excellent.

ERIE On Apr. 6 the annual Management Night of the Erie Section was held at the Blue Room of the Erie Moose Club. Nyles V. Reinfeld, President of Executive Services, Cleveland, Ohio, spoke on "Linear Programming."

GEORGIA On Apr. 15 the Georgia Section held its spring meeting in the Hotel Demsey, Macon, Ga. The dinner meeting was addressed by Capt. Carter A. Printup, USN, Commanding Officer, Naval Ordnance Plant, Macon, Ga. A series of papers were presented by the following: Ralph Coon, Fulton Bag and Cotton Mills, presented a paper on "Summary-graphs;" W. B. Simmons, St. Joe Paper Co., spoke on some practical applications of rank order statistics; and Donald C. Flischel, specialist in mathematical statistics, Wright-Patterson Air Force Base, Dayton, Ohio, presented a talk on the Air Force quality program in procurement, supply, and maintenance.

HUNTSVILLE The Huntsville Section met Apr. 21 in the Longfellow Room of the Huntsville Public Library. David S. Chambers was the speaker and the topic of his talk was "On the Design of a Statistical Test."

Mr. Chambers, Associate Professor of Statistics at the University of Tennessee, also acts as a quality control consultant for various industries located in Tennessee. Mr. Chambers is a senior member of ASQC.

INDIANAPOLIS On Apr. 19 the Indianapolis Section held the monthly afternoon technical session with C. R. McDowell, Allison Division, GMC (and member of the Indianapolis Section) as the speaker. His subject was "The Application of Work Sampling or Ratio Delay."

The regular monthly evening meeting was addressed by Edgar King, Staff Research Statistician and Education Chairman of the Indianapolis Section. His subject was "Practical Problems in Experimental Design."

INDIANAPOLIS Columbus Sub-Section The May meeting of the Columbus Sub-Section was addressed by Arthur H. Benscoter, quality manager, A. O. Smith Corp. of Kankakee, Ill. He spoke on quality control techniques used in his plant. He stated that the results of the techniques were better quality, dollar savings, and greater customer satisfaction.

INDIANAPOLIS Lafayette Sub-Section The Lafayette Sub-Section held a meeting on Apr. 27 in Room 220, Biological Annex, Purdue University. The speaker was William C. Vissing, chief of statistical quality control, Eli Lilly Co. His subject was "Quality Control in a Pharmaceutical Organization."

The meeting was dismissed early to enable all to see and hear Dr. Irving Burr give a talk on quality control over one of the local TV stations.

KANSAS CITY Approximately forty members attended the April dinner meeting. After the dinner hour and business session, during which officers for the coming year were elected, the meeting was turned over to the speaker of the evening, R. F. Ede. Mr. Ede, process metallurgist with United

States Steel, used as a subject "The Use of Analysis of Variance in Production Problems." Membership interest in the subject was indicated by the many questions which were directed to Mr. Ede after his talk.

LINCOLN The highlight of the April meeting of the Lincoln Section was a fine presentation on optical gaging given by Ted Froeberg, Optical Gaging Products, Inc. Mr. Froeberg supplemented his talk with the use of slides and a 16mm sound picture.

LOS ANGELES The April meeting of the Los Angeles Section was held jointly with the Instrument Society of America. As usual it was a huge success and it is developing more and more into an annual affair. The theme of this meeting was "Guided Missile Reliability" and was ably discussed by Bill Baker, Douglas Aircraft Corp., Robert E. Talmo, Winanko Engineering Co., and Dr. R. R. Carhart, Lockheed Missile Division.

MICHIGAN During April the Michigan Section held two sessions covering a wide range of quality control applications. Irwin A. DeGrote of our section presented a paper entitled "Quality Control by Statistical Methods, Applied to Foundry and Machining Operations." His talk presented some new and refreshing approaches to automotive applications.

Our other session was presented by W. F. Buhl who spoke on office applications of quality control. This presentation offered many new areas of operation for those present, which included the regular quality control group as well as many office managers who were our guests.

MILWAUKEE Warren R. Purcell addressed the Milwaukee Section on Apr. 18. His subject on "Quality Pre-Control—A New Tool for Long and Short Runs" was both interesting and informative. His views on this subject are very practical and can be used by all.

NEW HAVEN The Apr. 12 meeting featured R. E. Burt, President of R. E. Burt Associates. As guest speaker Mr. Burt's fine presentation of the topic "Human Relations in Quality Control," was enthusiastically received by members and guests. A social hour prior to the dinner and meeting was enjoyed by the good attendance.

RICHMOND "Operations Research" was the subject of a talk by John H. Roseboom, Jr. at the Apr. 26 dinner meeting. Mr. Roseboom, operations analyst at the Operations Research Office, The Johns Hopkins University, presented in general terms a review of O. R., its start, growth, and applications.

SALT LAKE CITY On Apr. 28 a joint meeting was held with the American Society for Metals, Utah Chapter. Frank G. Norris, Wheeling Steel Corp., West Virginia, spoke on "The Control of Quality in the Production of Low Carbon Basic Open Hearth Steel."

SAN ANTONIO At a meeting of the San Antonio Section held Apr. 6, J. Y. McClure, a National Director of ASQC, presented the Charter to the local section. Receiving the Charter for the San Antonio Section was R. C. Mussey, section chairman.

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Speakers on the program for the evening were Charles Hanson, IBM representative, and James Coburn, quality control, Convair, Ft. Worth, General Dynamics Corp. The subject matter covered by these speakers concerned the use and application of business machines in quality control.

SCRANTON -WILKES-BARRE On Apr. 20 a panel discussion on receiving inspection was held at the Scranton YMCA and an enthusiastic discussion period followed. Serving on the panel were: Harold Dumoff, Carl Larson, and Robert A. Zingermann. A. L. Lambert was moderator.

SEATTLE Melville F. Pugh, general traffic supervisor of the Pacific Telephone and Telegraph Co. in Seattle outlined the problems connected with sampling in the control of quality of operator performance. This was an enlightening talk to those who are used to thinking of quality control solely in terms of manufacturing applications.

SOUTH BEND -MISHAWAKA At the May meeting slides supplied by the Ford Motor Co. formed the basis for an evening devoted to the problem of vendor certification. Andy Thomas, Bendix Products, narrated and conducted a question and answer period generated by the slides.

STATE UNIVERSITY OF IOWA Robert L. Storer of the Ordnance Ammunition Command, Joliet, Ill., was the speaker at the April meeting of the Quint-City Group. Mr. Storer discussed some of the quality control applications at the O.A.C., particularly acceptance sampling. Some of the quality control innovations used at Joliet were quite new to many of those present.

ST. LOUIS Mr. Guy G. Parkin, Minnesota Mining & Mfg. Co., presented a very interesting talk entitled, "Forecasting Sales and the Liaison with Purchasing, Scheduling, Production, Shipping and Inventory" at the April meeting of the St. Louis Section. In view of the many ties in the subject matter of Mr. Parkin's talk, many visitors were in attendance.

TENNESSEE Memphis Sub-Section On Apr. 29 the monthly meeting of the Memphis Sub-Section was held in the cafeteria of Plough, Inc.

Ralph E. Rinella, statistical quality control director, Procter and Gamble Defense Corp., Milan, Tenn., presented his talk on "The Technique and Application of Continuous Sampling."

UNIVERSITY OF WESTERN ONTARIO A dinner meeting of the University of Western Ontario Section was held on Apr. 7 at the Panel Room of the YMCA in Kitchener. The guest speaker was Dr. Carl E. Noble of the Kimberly Clark Corp. The subject was "Cost of Quality," quite an important one in these days of reduced costs and tight competition. The talk was very well received, proof of which was shown by the number of interesting questions asked during the question and answer period following the speech.

On Apr. 21 the Kitchener Group of the U.W.O. Section was treated to a most informative talk on SQC by Prof. Brant Bonner. The discussion was admirably illustrated by a number of "gadgets" designed by Prof. Bonner.

On Apr. 29 the London Group of the U.W.O. Section heard an interesting talk by R. Mackay of the Ford Motor Company of Canada. The subject was "Managing Quality," and it emphasized the importance of the shop foreman in carrying out a successful drive on quality improvement. The talk was illustrated by a film.

WESTERN MASSACHUSETTS The last technical meeting of the season for this section was held at the Mountain Laurel Restaurant, Apr. 12. The speaker was C. W. Carter of Rath and Strong Consultants. He spoke on "Pre-Control." Mr. Carter illustrated his talk through the use of slides and proved very effectively that this method of control in many instances is superior to the regular control chart technique and that the data collected can serve excellently the purposes of engineering study such as process capabilities, operator performances and machine to machine variations. Examples of the use of this control relative to continuous process industries was also discussed.

WORCESTER The meeting of the Worcester Section on Apr. 27 was Ladies Night. William Stockdale of Putnam, Conn. presented his color motion picture and commentary on "Adventure in Alaska." The meeting was well attended and was enjoyed by all present.

YOUNGSTOWN Dr. Max Astrachan gave an excellent talk on the "Application of Correlation Analysis to Statistical Data" before the Youngstown Section at a dinner meeting on Apr. 14 at the Mural Restaurant in Youngstown, Ohio. His talk covered simple correlation and the interpretation of results. Prior to the technical session a business meeting was held and the new officers were elected.

". . . CAST THEIR SHADOWS BEFORE"

GEORGIA The new schedule for meetings next fall of the Atlanta Group is almost complete. It will be released in late summer for meetings to be held on the last Tuesday of each month beginning in September.

HUNTSVILLE For the June meeting, the Huntsville Section will have a joint meeting with the Huntsville Section of the Institute of Radio

Engineers and a field trip to the General Electric Tube Plant in Anniston, Ala. is planned.

MILWAUKEE The Milwaukee Section will hold their annual business and social meeting at the Pabst Brewing Co. on Monday, June 20 at 7:30 p.m.

NEW HAVEN The Executive Committee of the section will continue to hold regular monthly informal committee meetings at committee members' homes to formulate plans for the 1955-56 season.

RICHMOND The June 28 meeting will be primarily social with all wives being invited to the dinner. A short business session will be held for the election of officers for the coming year.

"AS YE SOW . . ."

BALTIMORE Dr. A. J. Duncan of Johns Hopkins, a pioneer in the teaching of statistical quality control, recently moved to a much larger private office in Maryland Hall in line with his progression of ever teaching more and more students the subject of SQC. In the process of moving, the school officials were amazed at the man hours turned in by the maintenance department in moving Dr. Duncan's thousands of books. The collection includes almost every paper, article, thesis and book written on the subject of quality control.

GEORGIA E. G. Field and D. S. Holmes of the Atlanta Group are busy in the Georgia Section's Committee on Training Courses. Atlanta, Macon, Savannah, and other cities have requested weekly classes on basic quality control. Any outlines and information from other sections will be appreciated.

MILWAUKEE The second meeting of the Tenth Midwest Quality Control planning committee was held at the Hotel Sherman, Chicago, with all committee chairmen in attendance. Presiding at the meeting was Les Eichelberger who after receiving the various committee reports predicted that the 10th Midwest Conference to be held in Milwaukee on November 17 and 18, would be truly outstanding.

SOUTHERN CONNECTICUT The Southern Connecticut Section was officially recognized through yet another source this month with the story of the April meeting being carried in the Chamber of Commerce's publication, *Bridgeport Progress*. John E. Butler, staff member of the Chamber and Editor of *Bridgeport Progress* welcomed the organization through its executive committee to continued participation in local activities. The section's April meeting in the Stratfield Hotel entertained A. P. Deacon, professional engineer and quality control management consultant in foundry practice to Canadian industries. Mr. Deacon is from Ontario, Canada.

UNIVERSITY OF WESTERN ONTARIO A short training course in SQC was recently conducted at No. 12 Technical Service Unit of the Royal Canadian Air Force, Western Ontario, for the officers and men of the command and civilian inspectors attached. This course, given in two sections, is believed to be the first course in SQC given in the RCAF. Instructor for the course was Prof. Brant Bonner, University of Western Ontario Section.

WESTERN MASSACHUSETTS A general meeting of all past officers and newly elected ones will be held at a date to be announced during the last week of June. The purpose of the meeting will be three-fold centering around the problem areas of: how to increase membership; develop a statisfactory program for the 1955-56 season relative to technical meetings; and developing an advanced course in applied statistics with the cooperation of the Dept. of Massachusetts University Extension. This educational program will be on a college level with proper credit allowed.

"FOR MANY ARE CALLED, BUT FEW ARE . . ."

ALBUQUERQUE The following officers have been elected for the 1955-56 season: Chairman, A. F. Cone; Vice-Chairman, R. E. McClendon; Secretary, N. A. Bercaw; Treasurer, W. E. Rehbeck; Director, Council of Technical and Scientific Societies, K. E. Schroder and A. F. Hurford.

BUFFALO The annual election of Directors held on Apr. 25 resulted in Francis Davenport of the Rudolph Wurlitzer Co. and Alfred J. Kukla of the Aluminum Company of America being elected for three year terms.

The Executive Board met and elected the following officers for the 1955-56 season: Chairman, Dr. E. L. Sexton; Vice-Chairman, Robert W. Stohr; Secretary-Treasurer, Alfred J. Kukla. The new officers were installed at the May meeting.

BUZZARDS BAY Installation of officers for the 1955-56 season was a part of the May meeting of the Buzzards Bay Section.

These included: Chairman, Martin S. Lindenberg; Vice-Chairman-Membership Chairman, Norman R. Cobb; Secretary, Zaven Mikaelian; Recording Secretary, John Ruggles, Jr.; Treasurer, Allan Fraser; Program Chairman, Mrs. Mary Lindenberg; Education Chairman, Robert E. Cooper; and Publicity Chairman, Daniel Katz.

CLEVELAND The officers for the coming year are: Chairman, Dr. Fred C. Leone, Case Institute of Technology; Vice-chairman, Howard A. Schmidt, Aluminum Co. of America; Secretary, Paul Kuckuk, Weatherhead Co.; Treasurer, Ralph Notingham, Brush Electronics Co.

ERIE Officers for the Erie Section for 1955-56 will be: Chairman, Donald E. Broderick; Vice-Chairman, David F. Ferrell; Secretary, Dorothy Kriechbaum; and Treasurer, Richard S. Albitz.

GEORGIA The following officers were elected for the 1955-56 season: Chairman, J. N. Little; Vice-Chairman, J. T. Hubbard; Secretary, Sam T. Ray; and Treasurer, J. K. Kinkead.

INDIANAPOLIS The Indianapolis Section elected the following members for officers of the coming 1955-56 season: President, Claude Adair; Vice-President, William Vissing; Treasurer, Thomas Turner; and Secretary, Richard Shaw.

KANSAS CITY Officers of the Kansas City Section elected for the ensuing year are: Chairman, Hugh Springer; Vice-Chairman, Glennon M. Putthoff; Secretary, Vivian Crawford; and Treasurer, Ivan Davis.

MONTREAL The newly appointed officers of the Montreal Section for the coming year are: Chairman, R. H. Peck; Vice-Chairman, E. N. Beckett; Secretary, R. Vance Ward; and Treasurer, E. C. Longhurst.

NEW HAVEN At the regular monthly dinner meeting the officers for the coming season were elected. They are: Chairman, Robert I. Callahan; Vice-Chairman, Gerald J. McVay; Secretary, Arthur A. Stolzenberg; and Treasurer, Jack Bradshaw.

ST. LOUIS The St. Louis Section officers for 1955-56 will be as follows: President, Joseph Movshin; Vice-president, Milton Weiss; Secretary, William H. Von Meyer; Treasurer, Sam Houston.

SAN ANTONIO Newly elected officers of the San Antonio Section are as follows: Chairman, Emzy S. Garrett, Jr.; Vice-Chairman, Alex G. Nagy; Secretary, John E. Goodwin; and Treasurer, Joseph W. Harrison.

SOUTHERN CONNECTICUT At a meeting held in the Manufacturers Association Building in Bridgeport, Conn. the following officers were nominated for the 1955-56 season to serve the Southern Connecticut Section: Chairman, Edward Demers; Vice-Chairman, Eugene Grumman; Secretary, Ralph Minervino; and Treasurer, Edgar Dawes. The officers now finishing out their tenure of office have done an outstanding job and comments have been favorably received throughout the year on the manner in which the various projects have been handled. A report just given shows the section listed as No. 4 nationally and No. 1 in New England.

SOUTHERN TIER SECTION The Southern Tier Section unanimously elected the following officers for the 1955-56 season: Chairman, Thomas H. Johnston; Vice-chariman, Theodore N. Lemoncelli; Secretary, Gordon S. Lucas; Treasurer, Robt. E. Kiley; Director, W. E. Andrus, Jr.; Director, John Harley.

The members of the Southern Tier Section extend their congratulations to Rocco Fiaschetti, Ansco Film Corp., who was elected to a two-year term as National Director of the ASQC.

WORCESTER The officers for the coming year are as follows: Chairman, Arthur P. Pingalore; Vice-Chairman, Warren D. Arnold, Jr.; Secretary, P. Roy Leaman; and Treasurer, Francis Olson.

YOUNGSTOWN The new officers elected for the 1955-56 season are: Chairman, D. J. Ausmus; Vice-Chairman, Frank Jayne; Secretary, Charles Cooper; and Treasurer, W. J. Cain.

DIVISIONS

Chemical Division

The present officers, Divisional Councilors, and the nominees for the ensuing year met in Chicago and discussed the following:

1. National Convention Program for Montreal.
2. Chemical Division Conferences—"In Miniature" Meetings.
3. Publications.
4. Technical Sub-Committees.
5. Membership, Member Interest.
6. Divisional Council Activity.
7. Administration Problems.

Our program and activities for next year are, in many cases, solely dependent upon the response to the questionnaire sent out earlier to the membership. Of 372 copies mailed to the membership, 100 were received. We must realize that the action taken by the officers was guided by the wishes of less than 25 percent of the Division membership.

We have received numerous suggestions regarding our forthcoming Division Directory. The most prominent being that our Directory not only show the name and address of each member, but his (or her) title, organization, etc., as well. This is desirable indeed, but since we do not have this information for each member and cannot easily incorporate such information for the few cases where it is available, our Directory will consist of the mailing address of each individual as it appears in our records at the time the Directory is issued.

MEMBERSHIP CORNER

The summer lull in formal section activities offers the membership committee an opportunity to review unhurriedly its past year's work as well as an opportunity to take a second look at its program for the coming year. The year ahead is of especial importance, membership-wise, as the Society hopes to achieve a goal of ten thousand members by its tenth birthday.

In reviewing the past year's work, have you taken advantage of all the opportunities to secure new members? Are there any procedures which might be improved? One section, following the practice of adding the names of guests to its mailing list, has instituted what it is hoped will be a more effective procedure: the name of each guest will be given to a member for personal attention; the member will send him a notice of the meetings, possibly with a note attached; he will also phone the prospective member about meetings, or call on him in person.

It may be profitable for the membership committee to review the names of non-members on the mailing list—not only may it permit getting rid of a lot of dead wood, but it may also reveal some situations worth following up. If a person has attended meetings but has not taken out membership, his employer might be approached on the basis that "here is an employee who is sufficiently interested in quality control to attend meetings—shouldn't the employer take advantage of this interest and see that he becomes a member of the Society?"

During the summer lull the membership list can be studied to see if there are any companies in the area served by the section which should be, but are not represented in the section membership. Personal calls on such firms can be very effective.

Are there persons whom members of the committee have intended contacting but haven't done so due to "pressure of business?" The summer period, with its slower tempo of business activity, should prove to be a good time to "catch up." Also, the summer may afford time to initiate new projects and have them well under way by fall. Why not take advantage of these opportunities?

SIGNIFICANT DIFFERENCES

William A. MacCrehan Award. This section of the October 1954 News Supplement reported briefly on a prize sponsored by W. A. MacCrehan and given by New York University.

Recently we received by direct inquiry to Bill more details on this award. It is called the "William A. MacCrehan Award," and is personally supported by Bill who was formerly on the faculty of New York University and who assisted in pioneering the establishment within the Industrial Engineering Department there a program of courses in engineering statistics and quality control.

Each year the Engineering Faculty of the Department of Industrial Engineering at NYU decides through a careful, selective process which student has shown the most promise in the field of engineering statistics and quality control. The award is open to students in both the day school and evening division program, and competition is keen. The awardee receives a two-year mebership in ASQC.

As Bill puts it in responding to our inquiry, "these courses are given in the senior year, and this award gets the fellow off on the right foot of being an ASQC member when he enters industry. My desire in giving this small award is only to foster a recognition of the importance of dealing with data in a sound manner—we spend years teaching the fellows how to collect data but only a two semester course in properly analyzing the meaning of data."

Edward B. Courtney has been appointed as field engineer in Metropolitan New York and Connecticut areas by Hunter Spring Co., Lansdale, Pa. Courtney has been Inspection Foreman at Hunter for the past five years and in that position attended special courses in statistics and quality control at Temple University. His experience of 21 years in Hunter's production, inspection and quality control departments will be helpful to users of springs, stampings, wire forms, and the company's new special spring products.

William B. Harrell, Secretary of the Huntsville Section, has accepted a new position with the government as an analytical statistician of the Acceptance Inspection Branch, Redstone Arsenal, Huntsville, Ala. He was previously with the Thiokol Chemical Corp.

Robert Fitzgibbons, well known for his vendor rating articles has been promoted to a section head at the Bendix Products Division of Bendix Aviation Corp. Since Bob has been an active member in local section activities, this transfer is Baltimore's loss and Mishawaka's gain.

Elbert Terhune, past president and member of the Indianapolis Section advisory committee, has terminated his employment with the U. S. Naval Ordnance Plant at Indianapolis and is now with Power Equipment Co., Galin, Ohio, as a sales engineer.

The Richmond Section is again grateful to Dr. Gerbert Arkin for the generous giving of his time to instructing a small group in some of the advanced techniques of SQC. Dr. Arkin, dean of the school of statistics, College of the City of New York and industrial consultant, is devoting one evening a week in Richmond (as he has in the past two years) to the teaching of analysis of variance and design of experiments. His interest and assistance has certainly done much to raise the level of application of SQC in the Richmond area.

Dr. Grant I. Butterbaugh, Fellow member of the Seattle Section and District 16 Representative, also General Editor of the *International Journal of Abstracts on Statistical Methods in Industry* will attend the 29th biennial Congress of International Statistical Institute in Rio de Janeiro June 24 to July 2 on invitation of the ISI and the Brazilian Government.

The Society notes with deepest regret the passing of Neal Dow Becker, May 18. Mr. Becker had been a member-at-large since 1949. At the time of his death, Mr. Becker was serving as chairman of the board of the Intertype Corporation, New York City.

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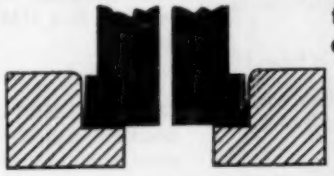

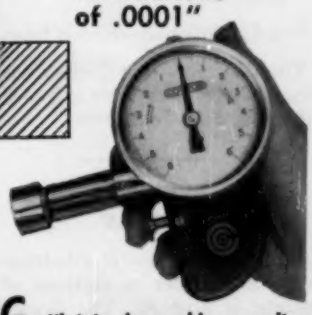

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TEXTILE CONFERENCE

An industrial training conference on Textile Statistical Quality Control and Uniformity Measurements was held Apr. 18-22, at the Institute of Textile Technology, Charlottesville, Va., under Conference Leadership of Dr. E. J. Bernet, N. L. Enrick, and several other staff members and graduate students at the Institute.

Special time was allotted to new electronic measuring instruments, particularly the Brush Uniformity Analyzer. Developed at the Institute, it automatically records the variation in textile strands passing through two measuring plates and electronically computes the Average Percent Range. Research has shown that since distributions within textile strands are usually skewed, the average range characterizes quality better than the standard deviation.

REPORT OF EXAMINING COMMITTEE

The Examining Committee has just announced the election of the following members of the Society to the grade of FELLOW:

Name	Section
Alvin G. Brooks	Chicago
Henry J. Jacobson	Chicago
Harry W. Poole	Lansdale
R. Shaw Goldthwaite	Rhode Island
Roy G. Volkman	Rhode Island

The Officers of the Society wish to congratulate these members for achieving this distinction.

OUR EVER-GROWING FAMILY

With the addition of Chattanooga, the total number of sections of the Society is now 85. The Society is proud to extend a most cordial welcome to the new section.

85—Chattanooga (Dist. 15)

Chairman: Richard S. Bingham, Jr.
Atlas Powder Co.
Chattanooga

Vice Chairman: Walter H. Corwin
Brock Candy Co.
Chattanooga

Secretary: William Whigham, III
E. I. DuPont de Nemours & Co.
Chattanooga

1955-56 YEARBOOK ANNOUNCEMENT

This is a reminder that the personal information cards sent to the membership in April are due back at the New York office by June 30. If you haven't already returned your card, please fill it in and send it off today so that our yearbook will be as accurate as possible.

Also—the dues bills which will be mailed out soon will include a post card on which you can request a copy of the new yearbook. Copies will *not* be sent automatically to each member; it is necessary that you return this card if you want a copy. Be sure to return it right after you get it, because the printing order will be based on the number of cards returned to New York.

POSITIONS WANTED

Address all replies to box number references to: American Society for Quality Control Room 563, 50 Church Street, New York 7, N. Y.

INDUSTRIAL ENGINEER desires position in management of SQC. B.S. in mathematics; minors: physics, education. Masters degree in Industrial Engineering; thesis: statistical methods in work measurement. Three years industrial experience in SQC techniques. Prefer medium sized company in midwest. Please reply to Box 11M1 at the above address.

QC ENGINEER desires supervisory position. Age 28; four years experience as supervisor controlling packaging equipment by SQC methods. Have also developed and implemented production efficiency methods. B.S. degree, pre-engineering and majored in statistics for QC. Please reply to Box 11M2 at the above address.

POSITIONS AVAILABLE

Address all replies to box number references to: American Society for Quality Control Room 563, 50 Church Street, New York 7, N. Y.

GLASS CONTAINER MANUFACTURER has an opening for a young man in quality work. Experience desirable but not essential. Age 25 to 30. Reply in confidence giving background in detail and salary expected. Please reply to Box 11Z1 at the above address.

CONSULTING SERVICES

Responsibility of the American Society for Quality Control, Inc., for Consulting Services advertising is limited to certification that advertisers hold the grade of membership in the Society stated in their advertisements. Qualification requirements for the several grades of membership are set forth in the Constitution of the Society.

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June 1955 Vol. XI, No. 9, Part III

Special Report of the Standards Committee
Submitted to Paul A. Robert, President, ASQC
March 4, 1955

The Standards Committee is pleased to submit herewith proposed ASQC Standard A2 on Definitions and Symbols for Acceptance Sampling.

In approaching the subject of standard definitions and symbols, such as may be used in the ASQC official publication, Industrial Quality Control, the Standards Committee has felt it desirable to consider separately the terms used in different categories. The first subject chosen, Control Charts, was covered in ASQC Std. A1-1951. The appended document relates to a second subject, Acceptance Sampling, and has the unanimous approval of the Standards Committee.

The duties of the Standards Committee, as given in the president's letter of instruction dated October 1, 1947, may be summarized as follows:

> "The Standards Committee is authorized to study and select those symbols, concepts, nomenclature, procedures and other matters which it feels the Society would find advantage in standardizing and to make recommendations to the Board of Directors for such standardization by the Society."

Accordingly the Standards Committee wishes to recommend to the Board of Directors that the Proposed Standard Definitions and Symbols for Acceptance Sampling submitted herewith be standardized by the American Society for Quality Control.

It is respectfully suggested, however, that before official action is taken with respect to this material, arrangements be made to send copies to each of the sections of the Society so that all may have an opportunity to submit comments before final approval.

ASQC Standards Committee

I. W. Burr
W. E. Deming
E. L. Grant
R. E. Wareham
H. F. Dodge, Chairman

Ed. Note: This proposed Standard A2 is being distributed to the membership of ASQC, as per the Standards Committee suggestion, to invite comments before it is accepted and published in final form. Please direct your suggestions to: Harold F. Dodge, Chairman ASQC Standards Committee, c/o Bell Telephone Laboratories, 463 West Street, New York, N. Y.

Proposed ASQC STD. A2:

on

Definitions and Symbols for Acceptance Sampling

The following definitions relate particularly to one of the most common uses of acceptance sampling, namely, acceptance inspection of individual lots of product, comprising individual units. The general intent of the definitions, however, should be considered applicable also to the inspection of bulk material, to the acceptance or rejection of a process on the basis of sampling results, etc. The alternative to acceptance is termed "rejection" for the purposes of definition, although, in practice, the alternative may take some form other than out-right rejection.

Term	Symbol or Abbreviation	Definition
1. GENERAL TERMS RELATING TO ACCEPTANCE SAMPLING.		
1.1 Acceptance Sampling		The art or science that deals with procedures in which decisions to accept or reject lots or processes are based on the examination of samples.
1.2 Sampling Plan		A specific plan which states (a) the sample sizes, and (b) the criteria for accepting, rejecting, or taking another sample to be used in inspecting a lot.
1.3 Inspection Lot		A specific quantity of similar material, or a collection of similar units, offered for inspection and acceptance at one time.
1.4 Lot Size	N	The number of units in the lot.
1.5 Sample		A portion of material or a group of units, taken from a lot, which serves to provide information for reaching a decision regarding acceptance.
1.6 Sample Size	n	The number of units in the sample.
1.7 Single Sampling		Sampling inspection in which a decision to accept or to reject is reached after the inspection of a single sample.
1.8 Double Sampling		Sampling inspection in which the inspection of the first sample leads to a decision to accept, to reject, or to take a second sample and the examination of a second sample, when required, always leads to a decision to accept or to reject.

Term	Symbol or Abbreviation	Definition
1.9 Multiple Sampling		Sampling inspection in which, after each sample, the decision may be to accept, to reject, or to take another sample, but in which there is usually a prescribed maximum number of samples, after which a decision to accept or to reject is reached. NOTE: Multiple sampling as defined here is sometimes called "sequential sampling" or "group sequential sampling". The term "multiple sampling" is preferred.
1.10 Sequential Sampling		Sampling inspection in which, after each unit is inspected, the decision is made to accept, to reject, or to inspect another unit. NOTE: Sequential sampling as defined here is sometimes called "unit sequential sampling".
1.11 Curtailed Inspection		Sampling inspection in which, as soon as a decision is certain, the inspection of the sample is stopped. Thus, as soon as the rejection number for defectives is reached, the decision is certain and no further inspection is necessary. NOTE: Commonly a first sample is always completed for the purpose of estimating the process average.
1.12 Normal Inspection		Inspection in accordance with a sampling plan that is used under ordinary circumstances.
1.13 Reduced Inspection		Inspection in accordance with a sampling plan requiring smaller sample sizes than those used in normal inspection.
1.14 Tightened Inspection		Inspection in accordance with a sampling plan that has more strict acceptance criteria than normal inspection.
1.15 Process Average Quality		Expected quality of product from a given process, usually estimated from first sample inspection results of past lots.
1.16 Acceptable Quality Level	AQL	The maximum per cent defective (or the maximum number of defects per 100 units) which can be considered satisfactory as a process average.
1.17 Probability of Acceptance	Pa	Probability that a lot or process will be accepted.

	Term	Symbol or Abbreviation	Definition
1.18	Operating Characteristic Curve for Acceptance Sampling Plan	OC Curve	A curve showing the relation between the probability of acceptance and either lot quality or process quality, whichever is applicable.
1.19	Consumer's Risk		The probability or risk of accepting a lot, for a given lot quality or process quality, whichever is applicable, Usually applied only to quality values that are relatively poor.
1.20	Producer's Risk		The probability or risk of rejecting a lot, for a given lot quality or process quality, whichever is applicable. Usually applied only to quality values that are relatively good.
1.21	Average Sample Number	ASN	The average number of sample units inspected per lot in reaching a decision to accept or to reject.
1.22	Average Total Inspection	ATI	The average number of units inspected per lot including all units in rejected lots (applicable when the procedure calls for 100% inspection of rejected lots.)
2.	TERMS RELATING TO ATTRIBUTES SAMPLING PLANS		
2.1	Acceptance Number	Ac	The largest number of defectives (or defects) in the sample or samples under consideration that will permit the acceptance of the inspection lot.
2.2	Rejection Number	Re	The smallest number of defectives (or defects) in the sample or samples under consideration that will require the rejection of the inspection lot.
2.3	Average Outgoing Quality	AOQ	The average quality of outgoing product after 100% inspection of rejected lots, with replacement by good units of all defective units found in inspection.
2.4	Average Outgoing Quality Limit	AOQL	The maximum average outgoing quality (AOQ) for a sampling plan.
2.5		P_{95}, P_{50}, P_{10}, P_{05}, etc.	Lot quality or process quality for which the probability of acceptance is 0.95, 0.50, 0.10, 0.05, etc. respectively for a given acceptance sampling plan.

NOTE: Of the entries in the second column, only N, n, Pa, P_{95}, P_{50}, P_{10}, and P_{05} are intended as mathematical symbols. All other entries are abbreviations for practical use.